MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level
Higher 2

CANDIDATE NAME: Lim

CENTRE NUMBER: S

INDEX NUMBER:

MATHEMATICS

9758/02

Paper 2

October/November 2024

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Solution served as a suggestion only

Solution served as a suggestion only

Section A: Pure Mathematics [40 marks]

1 Do not use a calculator in answering this question.

The complex number $-2+\mathrm{i}$ is denoted by z.

(a) Find the real numbers a and b such that $z = az^* + b$. [2]

$-2+i = a(-2-i)+b$

$= -2a+b - ai$

$\therefore a = -1$ $\qquad -2(-1)+b = -2$

$b = -4$

The complex number $1-3\mathrm{i}$ is denoted by w.

(b) Without using a calculator, evaluate $wz - \frac{w}{z}$. Give your answer in the form $c + d\mathrm{i}$, where c and d are real numbers. [4]

$w\left(z-\frac{1}{z}\right) = (1-3i)\left(-2+i-\frac{1}{-2+i}\times\frac{-2-i}{-2-i}\right)$

$= (1-3i)\left(-2+i-\left(-\frac{2}{5}-\frac{1}{5}i\right)\right)$

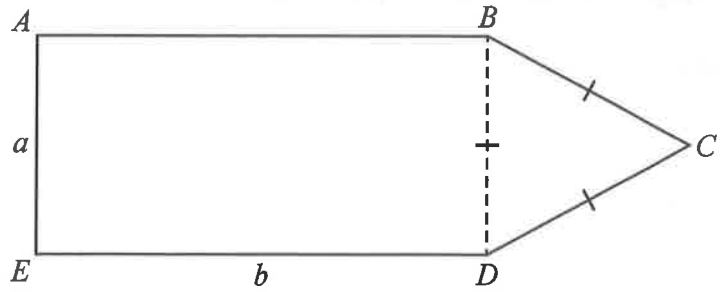
$= (1-3i)\left(-\frac{8}{5}+\frac{6}{5}i\right)$

$= -\frac{8}{5}+\frac{6}{5}i+\frac{24}{5}i+\frac{18}{5}$

$= 2+6i$

\#

2



A gardener designs a flower bed $ABCDE$ in the shape of a rectangle with an equilateral triangle on one of the shorter sides. Side AE is of length a m and side ED is of length b m (see diagram). The total perimeter of the flower bed is 20 m.

Find the maximum possible area of the flower bed, showing that it is a maximum value. Give your answer correct to 4 significant figures. [6]

$20 = 3a + 2b$ $\quad b = \frac{20-3a}{2}$ $\quad \because b > a$ then $10 - 1.5a > a$

$10 > 2.5a$

$a < 4$

Area : $A = \frac{1}{2}a^2\left(\frac{\sqrt{3}}{2}\right) + a\left(\frac{20-3a}{2}\right)$

sub in $a = 3.9999$

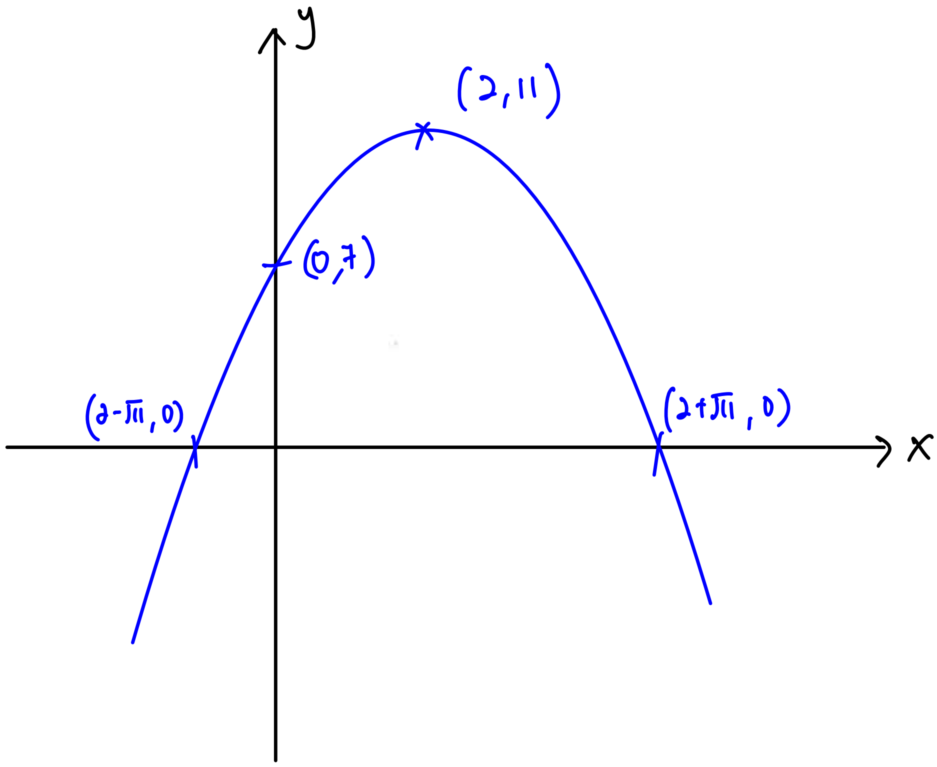
$A_{max} = 22.928 \approx 22.93$ (4sf)

3 The function f is such that $f(x) = -x^2 + 4x + 7$, for $x \in \mathbb{R}$.

(a) Find the range of f. [2]

$R_f : (-\infty, 11]$

(b) Sketch the graph of f, giving the exact coordinates of the points where the curve crosses the axes. [2]



$$x: \frac{-4 \pm \sqrt{16-4(-1)(7)}}{-2}$$

$$= \frac{-4 \pm 2\sqrt{11}}{-2}$$

$$= 2 \pm \sqrt{11}$$

The function g is such that $g(x) = \frac{2}{x} + 1$, for $1 \leqslant x \leqslant 10$.

(c) Explain how you know g^{-1} exists. [1]

$g(x)$ is a decreasing function for $1 \le x \le 10$.

Hence g is a 1-1 function and thus g^{-1} exists

(d) Find the value of $fg^{-1}(1.5)$. [3]

$\because \; g(4) = 1.5$

$fg^{-1}(1.5) = f(4) = 7$ //

4 The sum of the first n terms of the series T is given by $2n^3 - 8n^2 - 4n$.

(a) Find an expression for t_n, the nth term of series T. [2]

$$S_n - S_{n-1} = (2n^3 - 8n^2 - 4n) - (2(n-1)^3 - 8(n-1)^2 - 4(n-1))$$

$$= 2n^3 - 8n^2 - 4n - (2(n^3 - 3n^2 + 3n - 1) - 8(n^2 - 2n + 1) - 4n + 4)$$

$$= -8n^2 - 4n - (-6n^2 + 6n - 2 - 8n^2 + 16n - 8 - 4n + 4)$$

$$= 6n^2 - 22n + 6$$

The nth term of the series U is given by $u_n = 50n - 204$.

(b) Find the values of n for which $u_n = t_n$. [2]

n	$50n - 204$	$6n^2 - 22n + 6$
5	46	46
7	146	146

$n = 5, 7$

The nth term of the series V is given by $v_n = 3n + 16$.

(c) Find the smallest number greater than 100 that is in both series U and series V. [2]

n	$3n+16$	$50n-204$
60	196	
8		196

196

The nth term of the series W is given by $w_n = 3n^2 - 5n + 7$.

(d) **(i)** Explain why all the terms in series W are odd numbers. [2]

$3n^2 - 3n - 2n + 7$

$$\underbrace{\underbrace{\underbrace{3\overbrace{(n)(n-1)}^{①\text{ even}}}_{②\text{ even}} - \overbrace{2n}^{\text{even}}}_{③\text{ even}} + 7}_{④\text{ odd.}}$$

① product of 2 consecutive # is even

② product of even & odd # is even

③ differences between 2 even # is even

④ Sum of even & odd # is odd

(ii) Hence explain why series U and series W do not have any terms in common. [1]

Series U : $u_n = 50n - 204$

$= 2(25n - 102)$

All terms in series U are even

All terms in series W are odd.

$\therefore$ No common terms

5 **(a)** The graph of $y = f(x)$ intersects the x-axis at the point $(a, 0)$ and the y-axis at the point $(0, b)$.

(i) The graph of $y = f(x)$ is shown in Fig. 1. The scales on the x- and y-axes are the same.

Sketch the graph of $y = f^{-1}(x)$ on Fig. 1, labelling the intersections with the axes. [1]

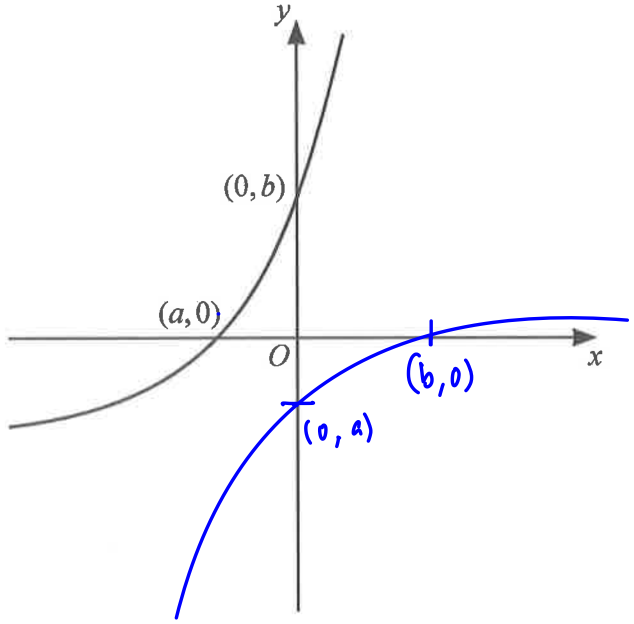


Fig. 1

(ii) The graph of $y = f(x)$ is also shown in Fig. 2. The scales on the x- and y-axes are the same.

Sketch the graph of $y = 2f^{-1}(x-1)$ on Fig. 2, labelling the intersection with the x-axis. [2]

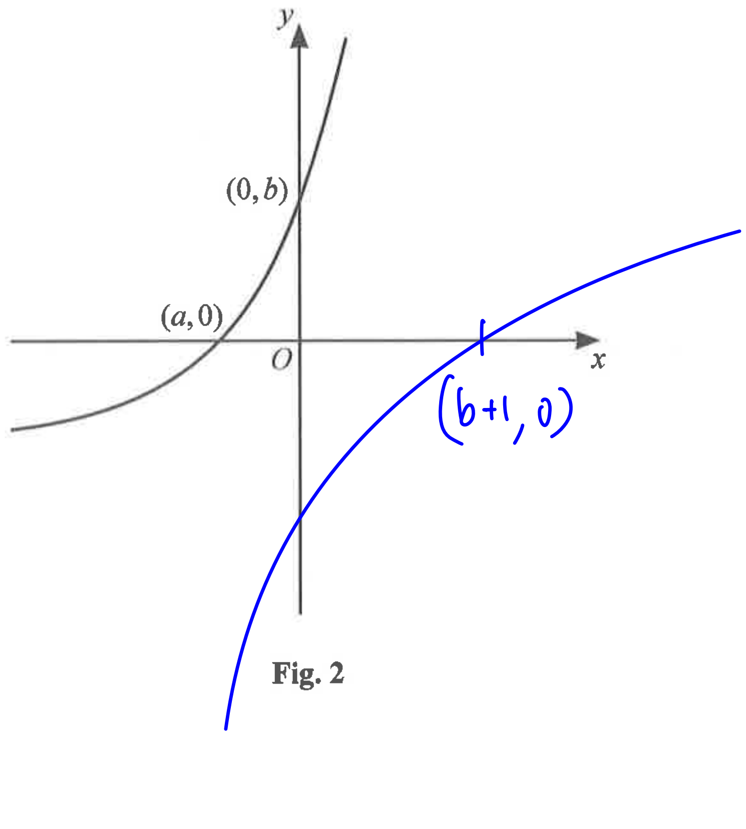


Fig. 2

(b) The graph of $y = \mathrm{g}(x)$ intersects the x-axis at the points $(s, 0)$, $(t, 0)$ and $(u, 0)$ and the y-axis at the point $(0, v)$.

(i) The graph of $y = \mathrm{g}(x)$ is shown in Fig. 3.1.

Sketch the graph of $y = |\mathrm{g}(x)|$ on Fig. 3.2, labelling the points where the graph intersects or touches the axes. [2]

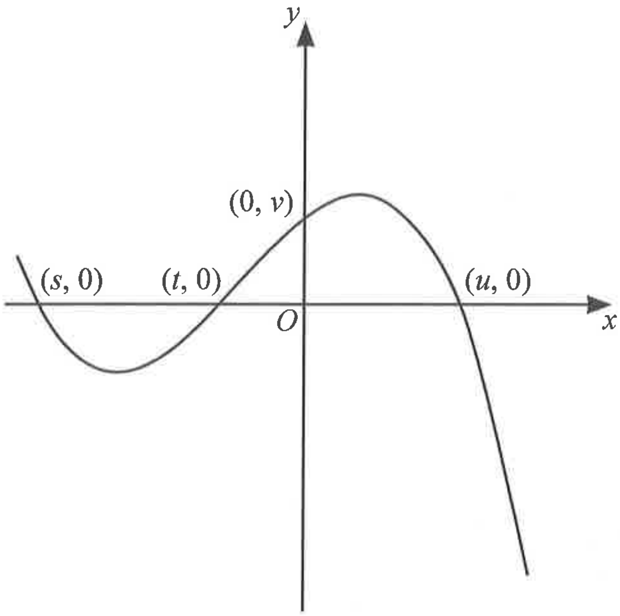


Fig. 3.1

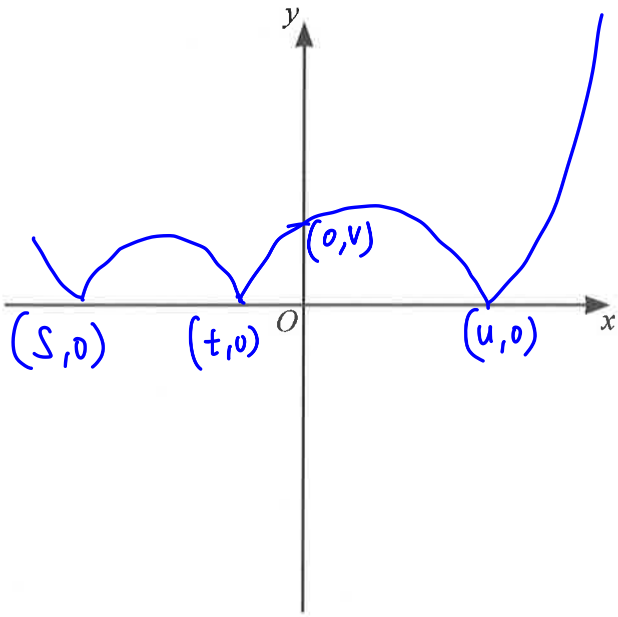


Fig. 3.2

(ii) The graph of $y = \mathrm{g}(x)$ is also shown in Fig. 4.1.

Sketch the graph of $y = \mathrm{g}(|x|)$ on Fig. 4.2, labelling the points where the graph intersects or touches the axes. [2]

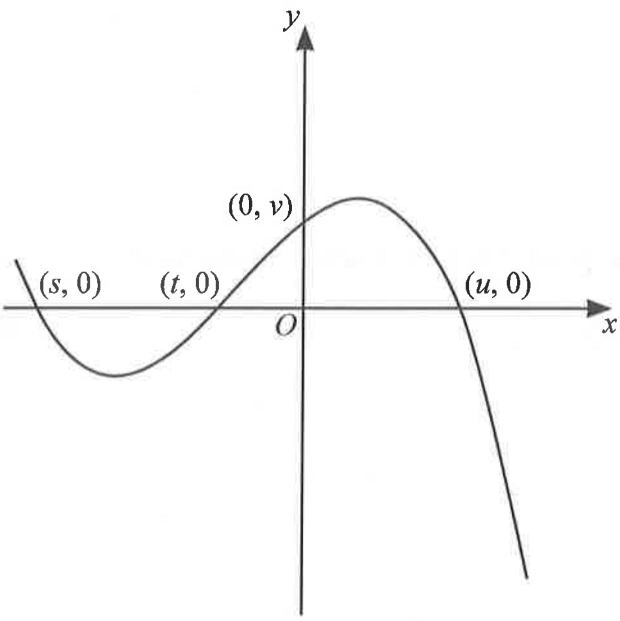


Fig. 4.1

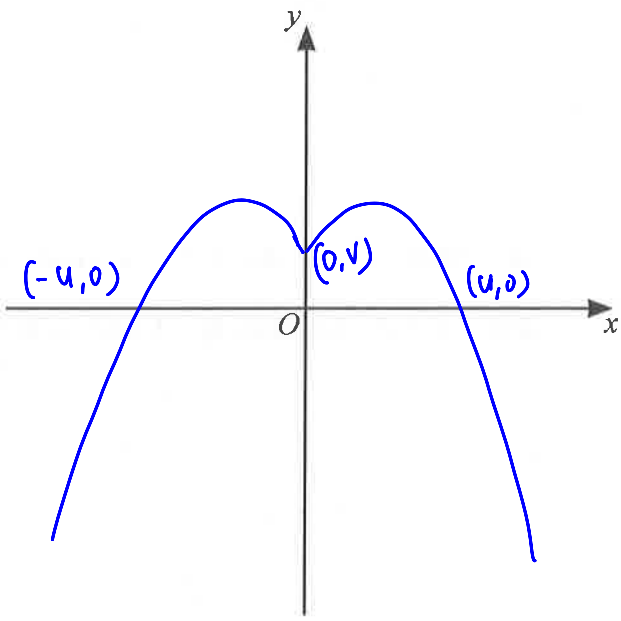


Fig. 4.2

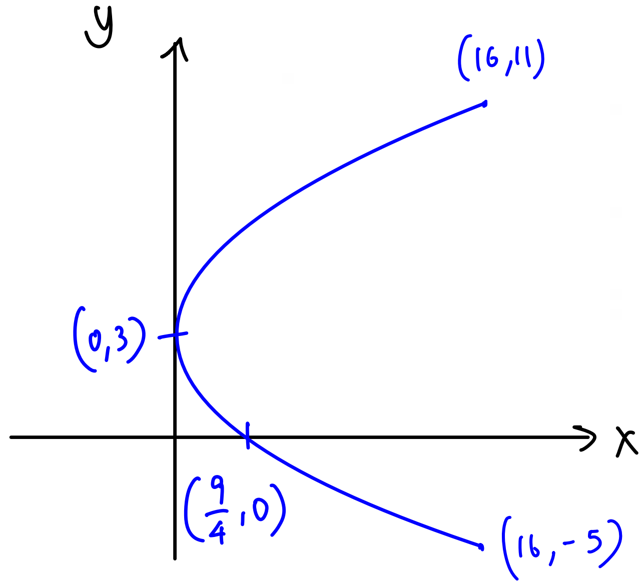
Solution served as a suggestion only

5 **[Continued]**

(c) The curve C has parametric equations

$$x = t^2, \qquad y = 2t + 3, \qquad \text{for } -4 \leqslant t \leqslant 4.$$

(i) Sketch the graph of C. [1]



(ii) Mark on your sketch the coordinates of the points where the curve C meets the axes. [2]

(iii) State the equations of any lines of symmetry. $y = 3$ [1]

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Solution served as a suggestion only

Section B: Probability and Statistics [60 marks]

6 A sports club has three categories of membership. There are 27 Youth members, 45 Adult members and 18 Senior members.

The club secretary wishes to find out the opinions of members of the club about the facilities offered. She gives a questionnaire to all the members of the club. She receives replies from 65 of the members.

(a) Explain whether the 65 members comprise a sample or a population. [1]

Sample as it is 65 out of the total of 90 members

The secretary decides to form a committee of members to discuss the results of her questionnaire.

(b) Explain an advantage of choosing a random sample of members for her committee. [1]

The results from her discussion will be unbiased.

The secretary decides that the committee will consist of 6 randomly chosen members, but there will be:

45

- at least 1 member from each category
- more Adult members than Youth members and
- more Adult members than Senior members.

(c) Find the probability that the committee contains more Youth members than Senior members. [4]

$$\text{Req Prob} = \frac{{}^{45}C_3 \cdot {}^{27}C_2 \cdot {}^{18}C_1}{{}^{45}C_3 \cdot {}^{27}C_2 \cdot {}^{18}C_1 + {}^{45}C_3 \cdot {}^{27}C_1 \cdot {}^{18}C_2 + {}^{45}C_4 \cdot {}^{27}C_1 \cdot {}^{18}C_1}$$

$$= \frac{13}{32}$$

7 (a) A, B and C are taking part in a game in which a prize is hidden in one of n identical closed boxes, where $n > 4$. First, A opens at random two boxes in an attempt to find the prize. If A fails to find the prize, B then opens at random two of the remaining boxes in an attempt to find the prize. If B fails to find the prize, C opens all the remaining boxes and finds the prize.

(i) Find, in terms of n, the probability that A finds the prize. [1]

$$2\left(\frac{1}{n} \times \frac{n-1}{n-1}\right) = \frac{2}{n}$$

(ii) Show that the probability that B finds the prize is $\frac{2}{n}$. [1]

P(A fail , B win)

$$= \frac{n-1}{n} \times \frac{n-2}{n-1} \times \left(\frac{1}{n-2} \times \frac{n-3}{n-3}\right) 2 = \frac{2}{n}$$

(iii) Find the range of values of n for which the probability that C finds the prize is more than 8 times the probability that either A or B find the prize. [3]

$$8\left(\frac{4}{n}\right) < 1 - \frac{4}{n}$$

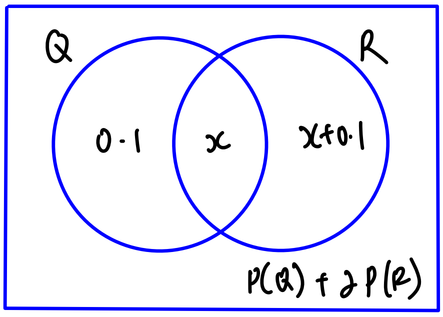
$$\frac{32}{n} < \frac{n-4}{n}$$

$$32 < n-4$$

$$36 < n \quad , \quad n \in \mathbb{N}$$

7 **[Continued]**

(b) The events Q and R are such that $\mathrm{P}(Q \cap R) = x$, $\mathrm{P}(Q \cap R') = 0.1$, $\mathrm{P}(Q' \cap R) = \mathrm{P}(Q)$ and $\mathrm{P}((Q \cup R)') = \mathrm{P}(Q) + 2\mathrm{P}(R)$. By using a Venn diagram, or otherwise, find the exact value of x. [3]



$$0.1 + x + x + 0.1 + 0.1 + x + (2(2x + 0.1)) = 1$$

$$0.3 + 3x + 4x + 0.2 = 1$$

$$7x = 0.5$$

$$x = \frac{1}{14}$$

8 Lee has a bird feeder on his balcony. Every morning he spends 20 minutes, while he eats his breakfast, counting the number of birds that visit the feeder. Over many months he has found that the mean number of birds visiting the feeder in this time interval is 17.3. When the bird feeder was damaged in a storm, Lee replaced it with a new bird feeder.

He suspects that the mean number of birds visiting the new bird feeder while he eats his breakfast has reduced. He decides to check this with a hypothesis test at the α% level of significance, where α is an integer. He records the number of birds, x, visiting the feeder in 20 minutes each morning for a random sample of n mornings.

(a) Explain whether Lee should carry out a one-tailed test or a two-tailed test. [1]

One tailed test as he is investigating whether there is a reduction

(b) State hypotheses for Lee's test, defining any parameters that you use. [2]

Null hypothesis $H_0: \mu = 17.3$

Alternative hypothesis $H_1: \mu < 17.3$

μ denotes population mean number of birds visiting the feeder.

Here is a summary of the data Lee collected.

$$n = 32 \qquad \Sigma x = 512 \qquad \Sigma x^2 = 8702$$

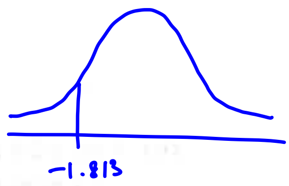
Lee carried out his test and concluded that the null hypothesis should be rejected.

(c) Calculate unbiased estimates of the population mean and variance of the number of birds, and determine the minimum possible value of the integer α. [3]

$\bar{x} = 16 \qquad s^2 = \frac{1}{31}\left(8702 - \frac{512^2}{32}\right) = \frac{510}{31}$

Under H_0, since n is large, by C.L.T., $\bar{X} \sim N\left(17.3, \frac{510/31}{32}\right)$ approx

Test Statistic : $z = \dfrac{16 - 17.3}{\sqrt{\frac{510/31}{32}}} = -1.813$



Rej H_0 if $z_{cal} < -1.813$

$\Rightarrow P(Z < -1.813) = 0.0349$

mini $\alpha = 4$

(d) State the conclusion to Lee's test in the context of the question. [1]

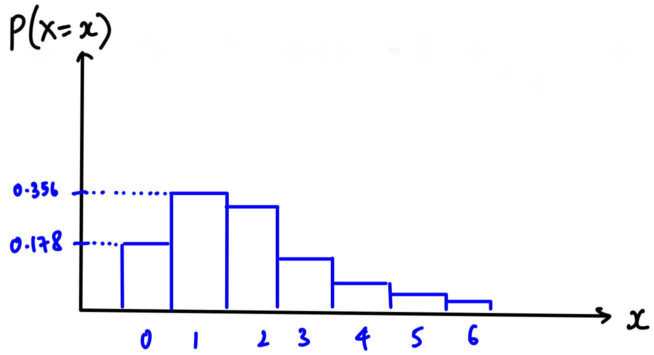
There is sufficient evidence at α% level of significance that the mean number of birds visiting the new bird feeder has reduced.

(e) Explain whether using the number of birds for each of 10 mornings would have been sufficient for Lee to carry out his hypothesis test. [1]

No. As he does not know the distribution, he will need a sufficiently large sample of n mornings, $n > 30$ so that by Central limit theorem, the sample mean number of birds visiting the feeder will follow normal distribution.

9 **(a)** The random variable X has the distribution B(6, 0.25). Sketch the distribution of X. [2]

$X \sim B(6, 0.25)$



(b) One of the jobs of a quality control operative in a food-processing factory is to check the quality of a random sample of 50 meat pies from the production line each day. She records the number of pies found to be 'unsatisfactory' each day for 80 days. Her results are shown in the table below.

Number unsatisfactory	0	1	2	3	4	5	6 or more
Frequency	57	9	6	4	3	1	0

Use the information in the table to estimate the probability that a randomly chosen pie is unsatisfactory. [2]

$$p = \frac{\text{No of unsatisfactory pies}}{\text{total \# of pies}} = \frac{0(57)+1(9)+\ldots 6(0)}{80(50)} = \frac{1}{80}$$

(c) One of the products made in the food-processing factory is the 'Frozen Cheesee Burger'. A fixed number of the burgers are tested each day and the number found to have insufficient cheese in them is denoted by Y.

(i) State, in context, two assumptions needed for Y to be well modelled by a binomial distribution. [2]

(1) Amount of cheese distributed in each burger is independent of other burgers

(2) Each burger has either sufficient cheese or insufficient cheese.

DO NOT WRITE IN THIS MARGIN

Assume now that Y has the distribution $B(120, 0.03)$.

(ii) Find the probability that, on a randomly chosen day, fewer than 3 burgers are found to have insufficient cheese in them. [1]

$Y \sim B(120, 0.03)$

$P(Y<3) = P(Y \le 2) = 0.2984$

≈ 0.298 #

(iii) For a randomly chosen period of 28 days, find the expectation of the number of days on which fewer than 3 burgers have insufficient cheese in them. [2]

$28 \times 0.298 = 8.356$

≈ 8 days.

(iv) Find the probability that, in a randomly chosen period of 28 days, more than 100 burgers are found to have insufficient cheese in them. [2]

Let X denote # of burgers with insufficent cheese

$X \sim B(3360, 0.03)$

$P(X>100) = 1 - P(X \le 100)$

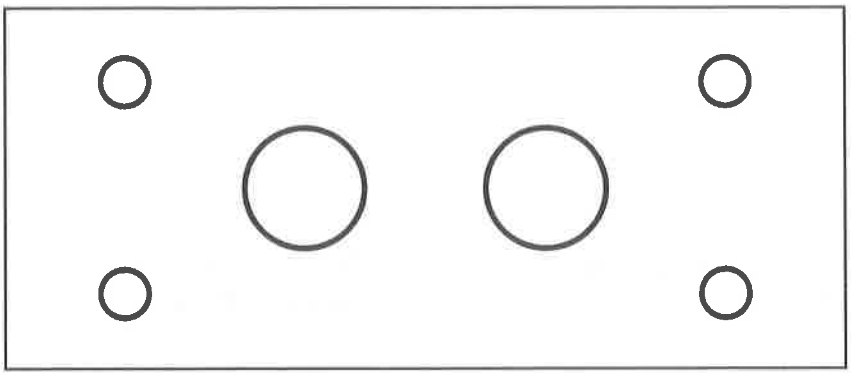
$= 0.5057$

≈ 0.506 #

Solution served as a suggestion only

10 In this question you should state the parameters of any distributions you use.

A company makes metal plates that can be used to fix fence panels onto posts. The metal plates have four small holes drilled into them for screws and two large holes drilled into them for bolts (see diagram).



Before the holes are drilled, the masses of plates, in grams, follow the distribution $N(200, 1.6^2)$.

(a) Find the probability that the mass of a randomly chosen plate before drilling is more than 197.5 grams. [1]

Let X denote mass of plates before drilling $X \sim N(200, 1.6^2)$

$P(X > 197.5) = 0.9409 \approx 0.941$

Drilling the holes reduces the mass of each plate by 5%. A production worker selects 8 of the drilled plates at random.

(b) Find the probability that at least 5 of these 8 plates have masses between 190 grams and 192 grams. [4]

Let $Y = 0.95X$ $\quad Y \sim N(190, 2.3104)$

$P(190 < Y < 192) = 0.4058776$

Let W denote # of metal plates with masses between 190 to 192 out of 8 plates $\quad W \sim B(8, 0.4058776)$

$P(W \geqslant 5) = 1 - P(W \leqslant 4)$

$= 0.1829$

≈ 0.183 #

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The drilled plates are sold in packs of 20 randomly chosen plates.

(c) Find the probability that the total mass of a pack of 20 drilled plates is less than 3805 grams. [3]

$Y_1 + \dots + Y_{20} \sim N(3800, 46.208)$

$P(Y_1 + \dots + Y_{20} < 3805) = 0.7689$

≈ 0.769 #

The manufacturer decides to sell 'Value' packs containing all the materials needed to fix fence panels onto posts. Each Value pack consists of 20 drilled plates together with the right number of screws and bolts to fit them; all of these are randomly chosen.

- The masses of screws, in grams, follow the distribution $N(10, 0.3^2)$. → 4 screws, not mass of each screw
- The masses of bolts, in grams, follow the distribution $N(44, 1.1^2)$. → 2 bolts

(d) Find the mass exceeded by just 5% of the Value packs. Give your answer to the nearest gram. You should ignore the mass of any packaging. [4]

Let S & B denote masses of screws & bolts

$S \sim N(10, 0.3^2)$ $\quad B \sim N(44, 1.1^2)$ $\quad Y_1 + \dots + Y_{20} \sim N(3800, 46.208)$

$T = S_1 + \dots + S_{20} + B_1 + \dots + B_{20} + Y_1 + \dots + Y_{20} \sim N(4880, 72.208)$

Let mass be m in grams

$P(T > m) = 0.05$

$P\left(Z > \dfrac{m - 4880}{\sqrt{72.208}}\right) = 0.05$

$\dfrac{m - 4880}{\sqrt{72.208}} = 1.64485$

$m = 4893.9$

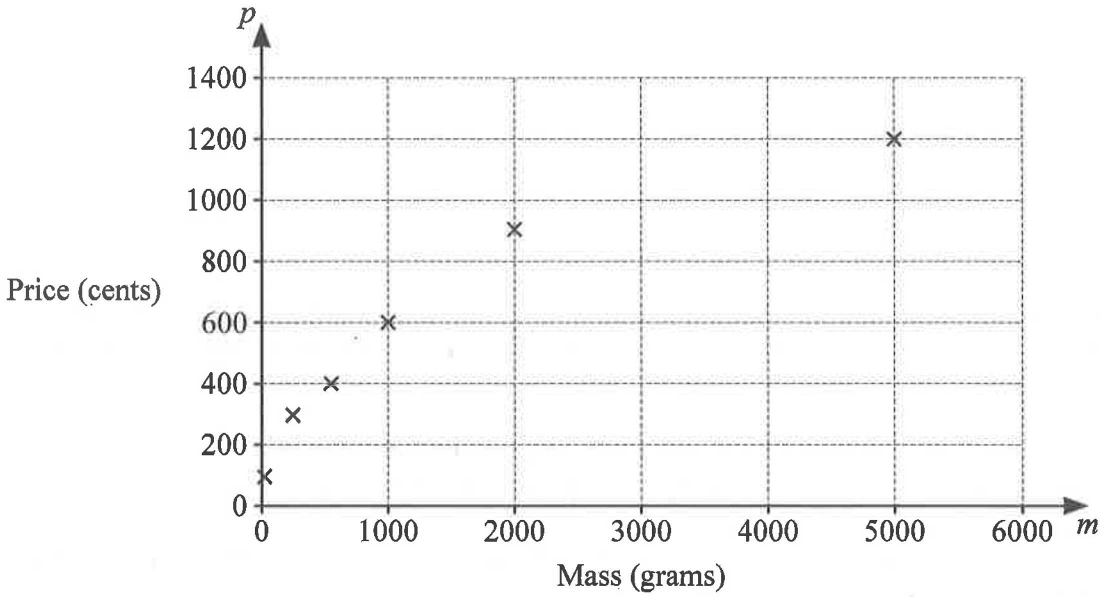
$\approx 4894\text{ g}$

Solution served as a suggestion only

11 **(a)** A certain brand of breakfast cereal is sold in a variety of different sized packs. Details of the mass of cereal in each pack, m grams, and the price, p cents, are given in the table below.

m	24	250	550	1000	2000	5000
p	100	300	400	600	900	1200

A scatter diagram for the data is shown below.



(i) Explain what the scatter diagram tells you about the relationship between m and p. [1]

non-linear r/s.

The following two models are proposed, where a, b, d, and e are constants.

$$p = a + b\ln m \qquad p = d + e\sqrt{m}$$

(ii) Determine which of these models gives the better fit to the data. State the values of the constants and the product moment correlation coefficient in this case. [4]

Model $p = d + e\sqrt{m}$ gives a better fit as $|r| = 0.9914$ is closer to 1

$p = 32.952567 + 17.2698\sqrt{m}$

$d \approx 33.0$

$e \approx 17.3$

(iii) A new pack containing 750 grams of cereal is introduced. Use the model you identified in part **(a)(ii)** to estimate the price of this pack, correct to the nearest 10 cents. Explain whether your answer is reliable. [2]

$p = 32.952567 + 17.2698\sqrt{750}$

$= 505.905 \approx 506$

Reliable, as $|r|$ is close to 1 and 750g is within data range.

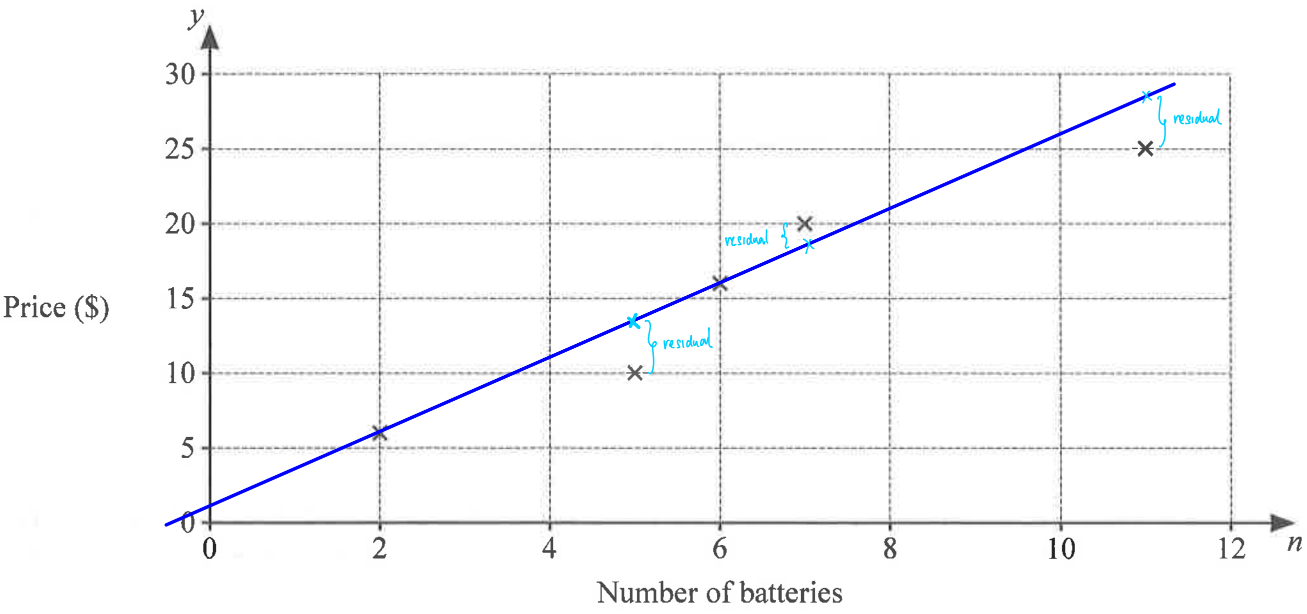
Solution served as a suggestion only

11 [Continued]

(b) Kai is investigating the relationship between the number, n, of a particular type of high-performance batteries in a pack and the price, $\$y$, of the pack she found in different stores. Her results are shown in the table below.

n	2	5	6	7	11
y	6	10	16	20	25

This information is shown in the scatter diagram below.



Kai decides to investigate whether the model $y = \frac{5}{2}n + 1$ is a good fit for this data.

(i) Draw the line $y = \frac{5}{2}n + 1$ on the scatter diagram above. [1]

(ii) For the model $y = \mathrm{f}(n)$, the residual for a point $(a,\ b)$ is $b - \mathrm{f}(a)$.

(A) Mark the residuals for the points on the scatter diagram above. [1]

(B) Explain why Kai should use the sum of the squares of the residuals rather than the sum of the residuals when assessing the fit of the model. [1]

$b - f(a)$ can be negative and summing them will give an erroneous result.

(C) Calculate the sum of the squares of the residuals for the line $y = \frac{5}{2}n + 1$. [1]

From G.C. 26.75

Kai's friend points out that other lines parallel to Kai's line can be drawn which are a better fit for the data.

(iii) Explain how the sum of the squares of the residuals for a line that is a better fit for the data differs from the sum of the squares of the residuals for the line $y = \frac{5}{2}n + 1$ found in part **(ii)(C)**. [1]

smaller.

(iv) Find the range of values of c for which the line $y = \frac{5}{2}n + c$ is a better fit for the data than the line $y = \frac{5}{2}n + 1$. [3]

$$(-1+c)^2 + (2.5+c)^2 + (-1+c)^2 + (-2.5+c)^2 + (2.5+c)^2$$

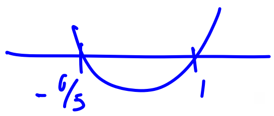
$$= 2(1-2c+c^2) + 2(6.25+5c+c^2) + (6.25-5c+c^2)$$

$$= 5c^2 + c + 20.75$$

for better fit $5c^2 + c + 20.75 < 26.75$

$5c^2 + c - 6 < 0$

$(c-1)(5c+6) < 0$



$\therefore -\frac{6}{5} < c < 1$

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