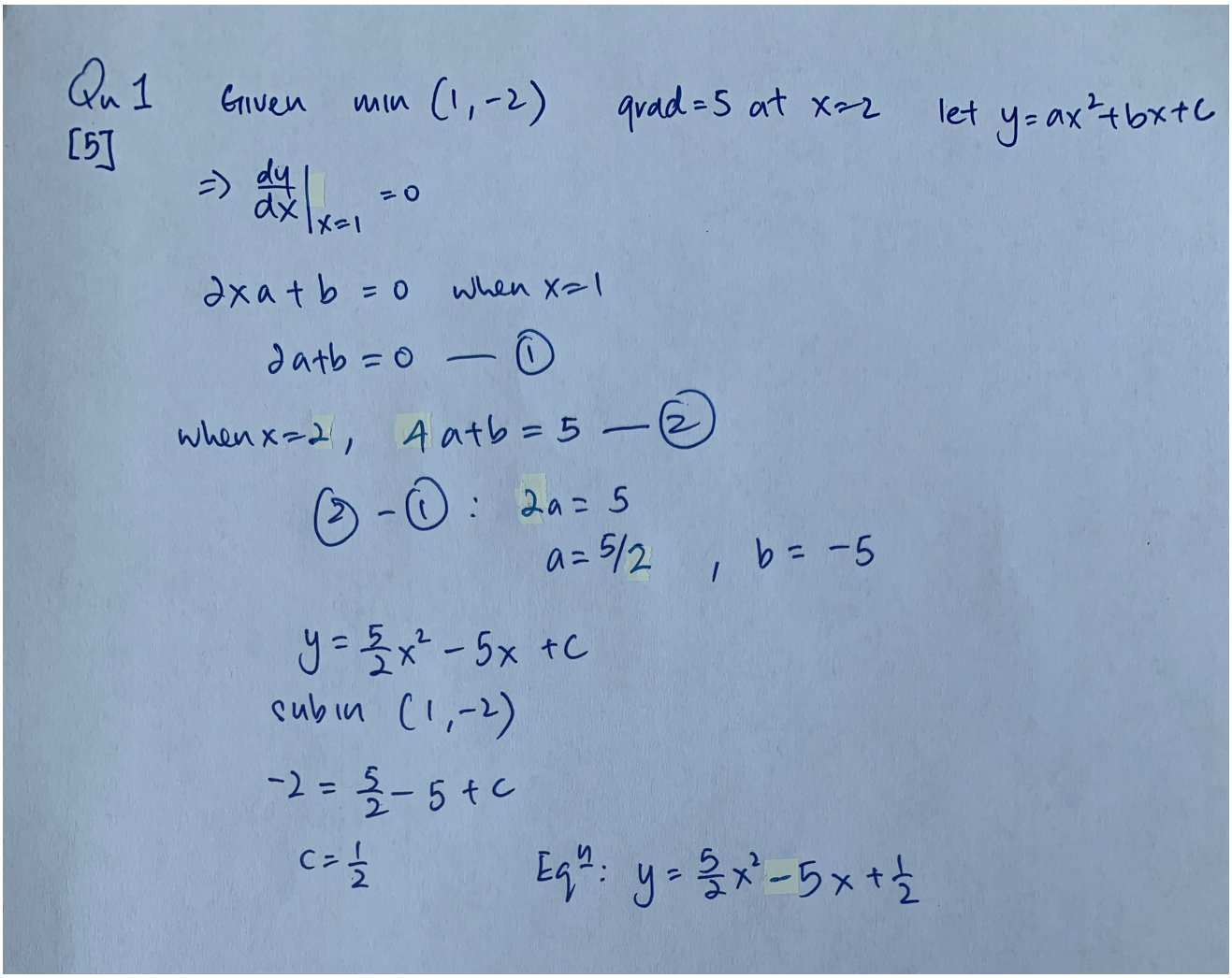
2020 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers



Qn 1 [5]

Given min $(1,-2)$ grad $=5$ at $x=2$ let $y=ax^2+bx+c$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 0$

$2\times a+b=0$ when $x=1$

$2a+b=0$ — ①

when $x=2$, $4a+b=5$ — ②

② − ①: $2a=5$

$a=5/2$, $b=-5$

$y=\frac{5}{2}x^2-5x+c$

sub in $(1,-2)$

$-2=\frac{5}{2}-5+c$

$c=\frac{1}{2}$

Eqn: $y=\frac{5}{2}x^2-5x+\frac{1}{2}$

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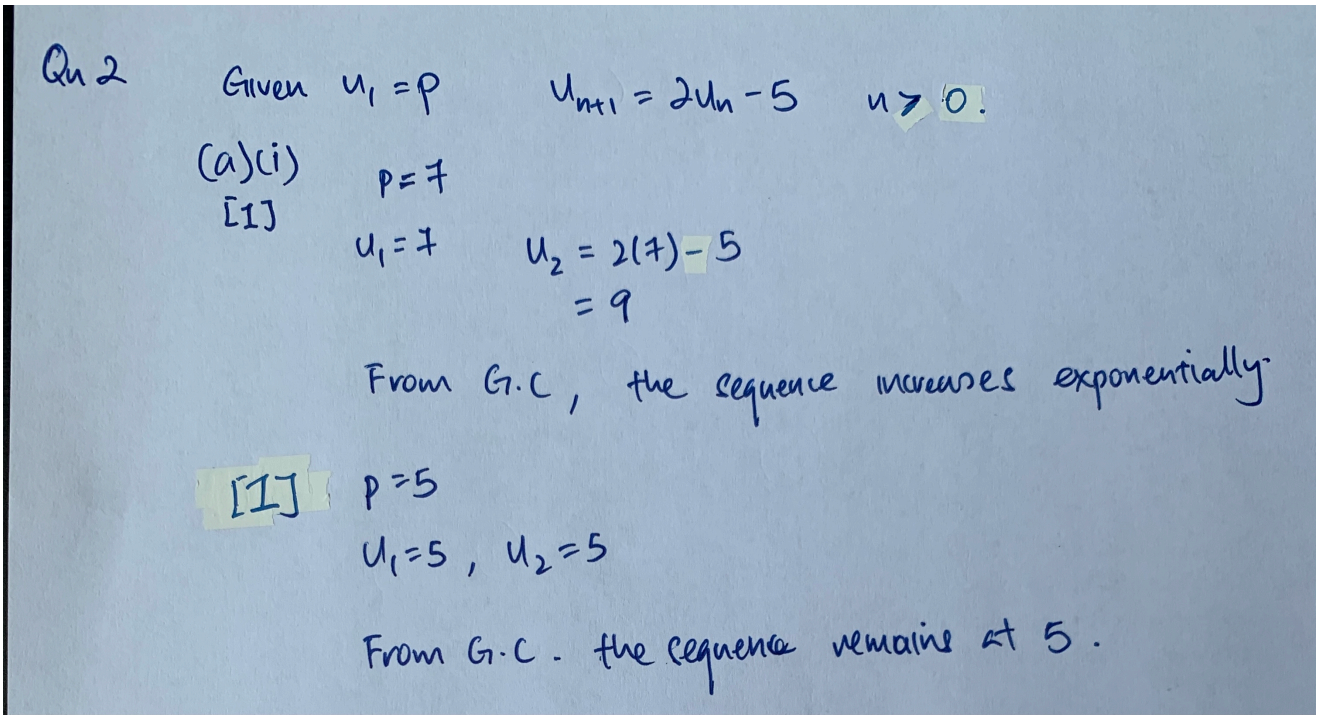
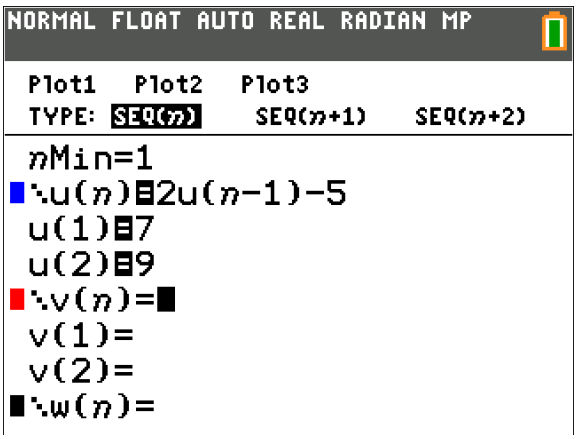
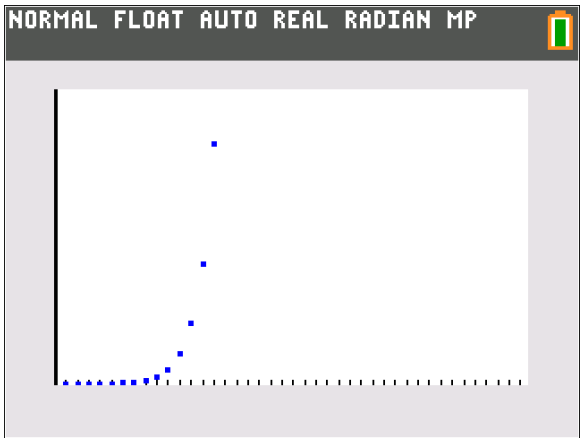


For Part 1





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a (ii) [2]

Given $u_5 = 101$

$101 = 2u_4 - 5$

$u_4 = 53$

$53 = 2u_3 - 5$

$u_3 = 29$

$29 = 2u_2 - 5$

$u_2 = 17$

$17 = 2u_1 - 5$

$u_1 = 11 = p$ //

Qn 2(b) (i) [3]

$v_1 = a \quad v_2 = b$

$v_{n+2} = v_n + 2v_{n+1} - 7 \quad n > 0$

Given $v_4 = 2v_3$

$\Rightarrow v_4 = v_2 + 2v_3 - 7$

$v_2 = 7 = b$ //

(ii) [1]

$v_5 = v_3 + 2v_4 - 7$

$= v_3 + 2(2v_3) - 7$

$= 5v_3 - 7$

$= 5(v_1 + 2v_2 - 7) - 7$

$= 5(a + 2(7) - 7) - 7$

$= 5a + 28$ //

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Qu 2 c (i) [2]

Given Σ 1st n term : $n^3 - 11n^2 + 4n$

$S_n = n^3 - 11n^2 + 4n$

$S_{n-1} = (n-1)^3 - 11(n-1)^2 + 4(n-1)$

$= (n^2 - 2n + 1)(n-1) - 11(n^2 - 2n + 1) + 4n - 4$

$= n^3 - n^2 - 2n^2 + 2n + n - 1 - 11n^2 + 22n - 11 + 4n - 4$

$= n^3 - 14n^2 + 29n - 16$

$T_n = S_n - S_{n-1} = 3n^2 - 25n + 16$ =

2c (ii) [2]

$S_m = m^3 - 11m^2 + 4m$

$S_3 = 3^3 - 11(9) + 12 = -60$

$m^3 - 11m^2 + 4m = -60$

From G.C : $m = 10, 3, -2$ (rej) (rej)

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Suggested Answers

AO studies
where transFormAtion begins

3 (i) [5]

$x = 3t^2 + 2 \qquad y = 6t - 1 \qquad t \geq \frac{1}{6}$

Given line N is normal at (14, 11)

$\frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 6$

$\frac{dy}{dx} = \frac{6}{6t} = \frac{1}{t}$

$x = 14 = 3t^2 + 2$

$t = 2 \quad \because t > 0$

$\left.\frac{dy}{dx}\right|_{x=14} = \frac{1}{2}$ Grad of normal : -2

Eqn $\quad y - 11 = -2(x - 14)$

$y = -2x + 28 + 11$

$y = -2x + 39$

$2x + y = 39 \qquad a = 2, \ b = 1, \ c = 39$

3 (ii) [4]

$2x + 0 = 39$

$x = 39/2$

Area: $\int_{25/12}^{14} y \, dx + \frac{1}{2}\left(\frac{39}{2} - 14\right)(11)$

$= \int_{1/6}^{2} (6t - 1)\, 6t \, dt + \frac{121}{4}$

$= \int_{1/6}^{2} 36t^2 - 6t \, dt + \frac{121}{4}$

$= \left[12t^3 - 3t^2\right]_{1/6}^{2} + \frac{121}{4}$

$= (96 - 12) - \left(\frac{12}{216} - \frac{3}{36}\right) + \frac{121}{4}$

$= \frac{2057}{18} \approx 114 \text{ units}^2$

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3 (iii) (a) [1]

Given 2-way stretch, 2 in x-dir, 3 in y-dir

Req Area : $\frac{2057}{18} \times 2 \times 3 = \frac{2057}{3}$ units^2

(b) [2]

$C: x = 3\left(\frac{y+1}{6}\right)^2 + 2$

sub in $x = \frac{1}{2}x$, $y = \frac{1}{3}y$

$D: \frac{x}{2} = 3\left(\frac{\frac{y}{3}+1}{6}\right)^2 + 2$

$\frac{x}{2} = 3\left(\frac{y+3}{18}\right)^2 + 2$

$\frac{x}{2} = \frac{(y+3)^2}{108} + 2$

$x - \frac{(y+3)^2}{54} = 4$ //

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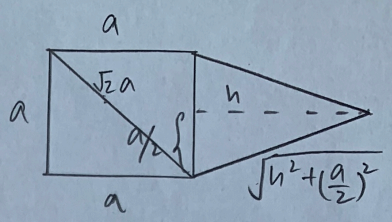
AO studies
where transFormAtion begins

(4) (i) [2]

$2h + a = 30$



$$-\left(\frac{\sqrt{2}a}{2}\right)^2 + h^2 + \left(\frac{a}{2}\right)^2 = H^2$$

$$-\frac{2a^2}{4} + \frac{a^2}{4} + \left(\frac{30-a}{2}\right)^2 = H^2$$

$$-\frac{a^2}{4} + \frac{900 - 60a + a^2}{4} = H^2$$

$$\Rightarrow H^2 = 225 - 15a \quad \# \text{ shown}$$

4 (ii) [5]

$$\text{Vol}: \frac{1}{3}(a^2)\left(\sqrt{225-15a}\right)$$

$$V^2 = \frac{1}{9}(a^4)(225-15a)$$

$$= \frac{225}{9}a^4 - \frac{15}{9}a^5$$

$$2V\left(\frac{dV}{da}\right) = \frac{900}{9}a^3 - \frac{75}{9}a^4$$

For max, $\frac{dV}{da} = 0 \Rightarrow 900a^3 = 75a^4$

$$12 = a \quad \because a \neq 0$$

$$V_{max} = \frac{1}{3}(144)\sqrt{45}$$

$$= 48\sqrt{45}$$

$$= 144\sqrt{5} \text{ cm}^3 \#$$

4 (iii)(a) [3]

Area : $4\left(\frac{1}{2}ah\right)$

$A = 2a\left(\frac{30-a}{2}\right)$

$= 30a - a^2$

$\frac{dA}{da} = 30 - 2a = 0$

$a = 15$

then $h = \frac{15}{2}$

5 (i) [1]

1 green, r red, 2r blue $r > 1$

0 point, 5 pts, 2 pts.

possible score:

Green, red		0
Green	blue	0
red	blue	10
red	red	25
blue	blue	4

∴ possible scores are, 0, 4, 10, 25

5 (ii) [7]

let X denotes Tina's score

$P(X=0) = \frac{1}{3r+1} \times \frac{3r}{3r} \times 2 = \frac{2}{3r+1}$

$P(X=4) = \frac{2r}{3r+1} \times \frac{2r-1}{3r} = \frac{2(2r-1)}{3(3r+1)}$

$P(X=10) = \frac{r}{3r+1} \times \frac{2r}{3r} \times 2 = \frac{4r}{3(3r+1)}$

$P(X=25) = \frac{r}{3r+1} \times \frac{r-1}{3r} = \frac{r-1}{3(3r+1)}$

x	0	4	10	25
$P(X=x)$	$\frac{2}{3r+1}$	$\frac{2(2r-1)}{3(3r+1)}$	$\frac{4r}{3(3r+1)}$	$\frac{r-1}{3(3r+1)}$

$E(X) = \frac{8(2r-1)}{3(3r+1)} + \frac{40r}{3(3r+1)} + \frac{25(r-1)}{3(3r+1)} = \frac{81r-33}{3(3r+1)}$

$= \frac{27r-11}{3r+1}$ = shown

$E(X^2) = \frac{32(2r-1) + 400r + 625(r-1)}{3(3r+1)} = \frac{1089r-657}{3(3r+1)}$

=

$\text{Var}(X) = E(X^2) - [E(X)]^2$

$= \frac{1089r-657}{3(3r+1)} - \left(\frac{27r-11}{3r+1}\right)^2$

$= \frac{(3r+1)(1089r-657) - (729r^2-594r+121)3}{3(3r+1)^2}$

$= \frac{3267r^2-1971r+1089r-657-2187r^2+1782r-363}{3(3r+1)^2}$

$= \frac{1080r^2+900r-1020}{3(3r+1)^2} = \frac{360r^2+300r-340}{(3r+1)^2}$ #

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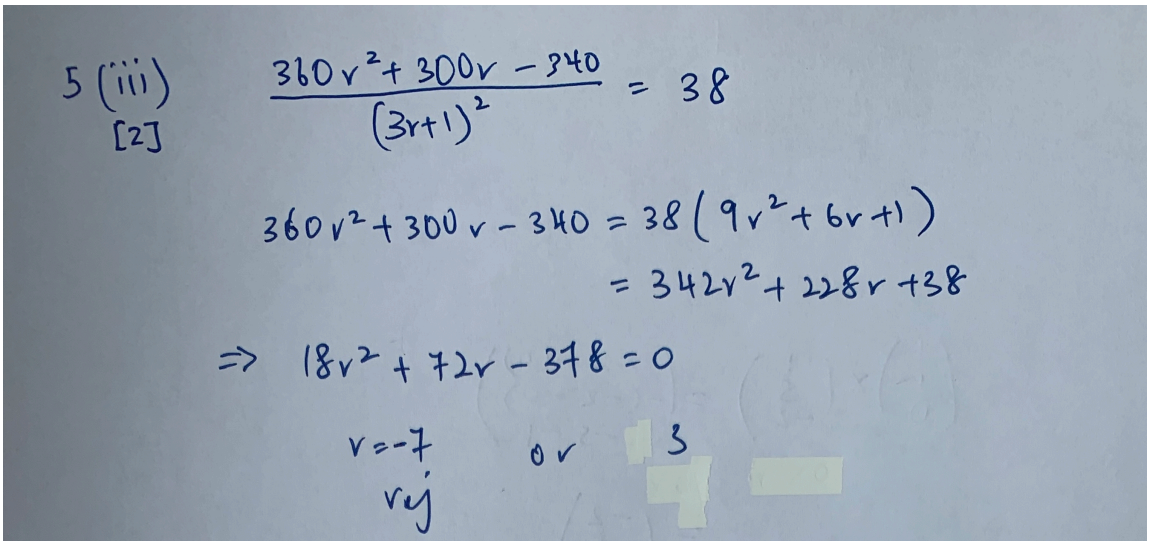


5 (iii) [2]

$$\frac{360r^2+300r-340}{(3r+1)^2} = 38$$

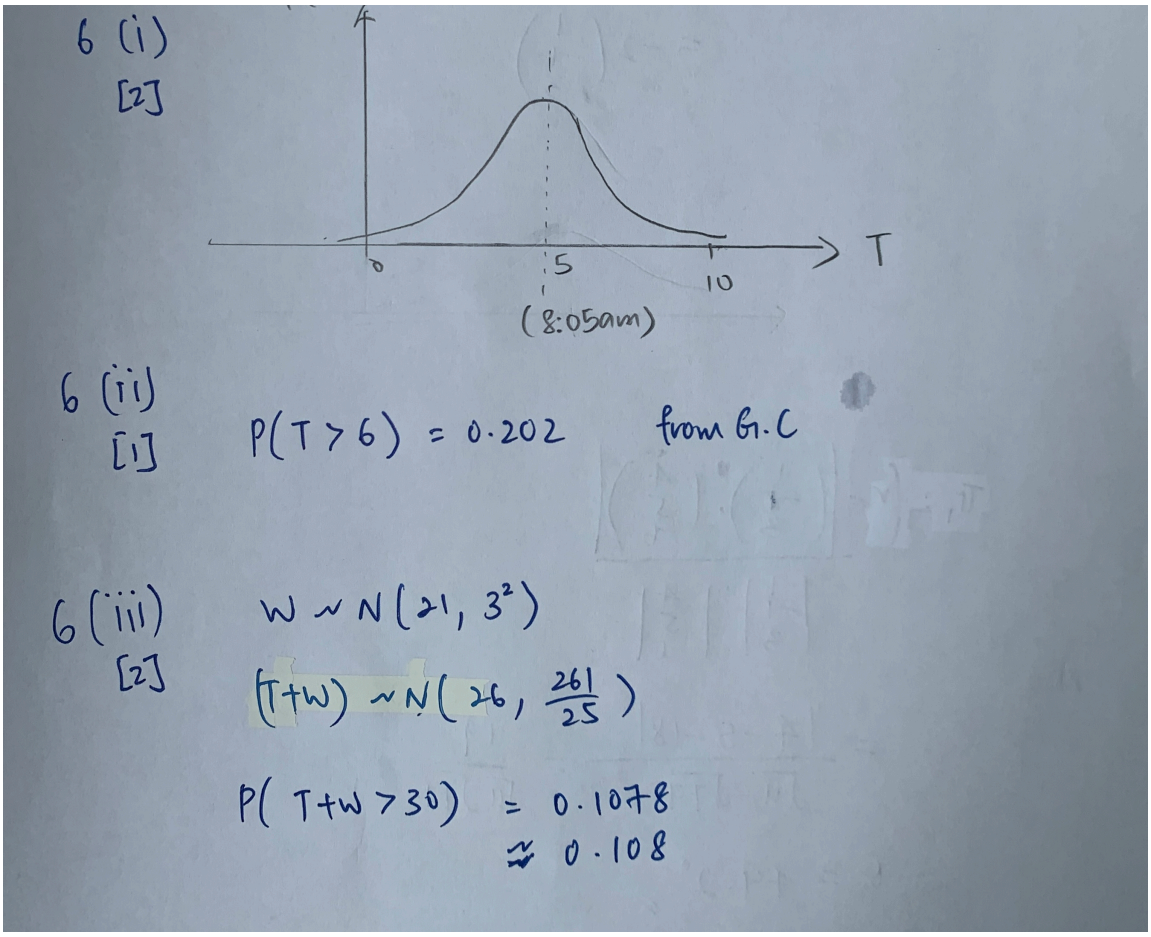
$$360r^2+300r-340 = 38(9r^2+6r+1)$$

$$= 342r^2+228r+38$$

$$\Rightarrow 18r^2+72r-378=0$$

$r=-7$ or 3

rej



6 (i) [2]

6 (ii) [1]

$P(T>6) = 0.202$ from G.C

6 (iii) [2]

$W \sim N(21, 3^2)$

$(T+W) \sim N\left(26, \frac{261}{25}\right)$

$P(T+W>30) = 0.1078$

≈ 0.108

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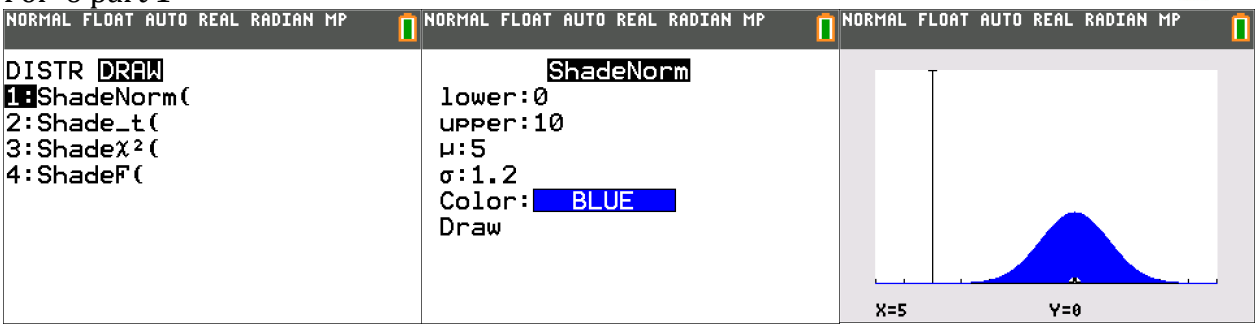
Suggested Answers

For 6 part 1



6 (iv) [5]

$T+D \sim N\left(24, \frac{936}{25}\right)$

$P(T+D > 30) = 0.16339$

$\therefore$ Prob late for work: $0.7(0.10778) + 0.3(0.16339)$

$= 0.124477$

Req Prob: $P(\text{late} \mid \text{fine})$

$= \frac{P(\text{late and fine})}{P(\text{late})}$

$= \frac{0.07546}{0.124477}$

$= 0.6062$

≈ 0.606 #

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(7) (i) $r = -0.848$ [2]

(ii) $a = 145.12 \approx 145$ [3]

$b = 9456.4 \approx 9460$

$r = 0.941$

(iii) do not fit. extrapolation. [1]

(iv) $d = -145.12 + 9456.4\left(\frac{1}{t}\right)$ [2]

$C = \frac{5}{9}F - \frac{160}{9}$

$\left(C + \frac{160}{9}\right)\frac{9}{5} = F$

Sub in

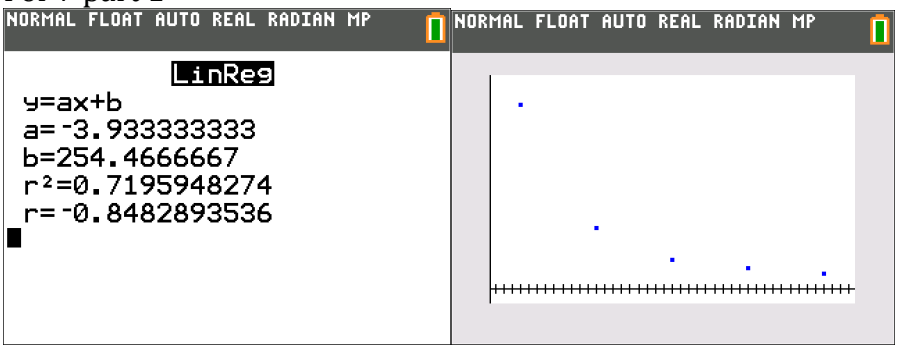
$d = -145.12 + 9456.4\left(\frac{9}{5}T + \frac{160}{5}\right)$

$= -145.12 + 17021.52T + 302604.8$

$= 17021.52T + 302459.68$

$\therefore d = 17000T + 302000$ //

For 7 part 1



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H2 Mathematics Paper 2 (9758/02)

Suggested Answers

For 7 part 2

8 (i) [2] 11 circles 17 rec

$P(R=1) = P(11 \text{ circles} + 1 \text{ rec})$

$= \frac{1 \times 17}{{}^{28}C_{12}} = \frac{17}{30421755}$

$P(R=2) = P(10 \text{ circles} + 2 \text{ rec})$

$= \frac{{}^{11}C_{10} \times {}^{17}C_2}{{}^{28}C_{12}} = \frac{1496}{30421755} > \frac{17}{30421755} = P(R=1)$

shown

8 (ii) [5]

$P(R=4) = \frac{{}^{11}C_8 \times {}^{17+r}C_4}{{}^{28+r}C_{12}}$

$P(R=3) = \frac{{}^{11}C_9 \times {}^{17+r}C_3}{{}^{28+r}C_{12}}$

$P(R=4) = 15(P(R=3))$

$({}^{11}C_8)({}^{17+r}C_4) = 15({}^{11}C_9)({}^{17+r}C_3)$

$\left(\frac{1}{5}\right)\frac{(17+r)(16+r)\ldots(1)}{(13+r)\ldots(1)\,4!} = \frac{(17+r)\ldots(1)}{(14+r)\ldots(1)(3!)}$

$\frac{1}{5}(14+r) = 4$

$14+r = 20$

$r = 6$ #

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AO studies
where transFormAtion begins

9 (i) [1] The probability of each pen being selected is equal and independent of each other.

(ii) [1] Let X be # of faulty pens in a box of 10 pens.

$X \sim B(0.06, 10)$

$P(X \leq 2) = 0.981$

(iii) [4] Prob rej : $1 - 0.98116$

$= 0.018837$

Let Y be # of boxes rejected out of 75

$Y \sim B(75, 0.018837)$

$P(Y > 3.75) = 1 - P(Y \leq 3)$

$= 0.053445$

≈ 0.0534 //

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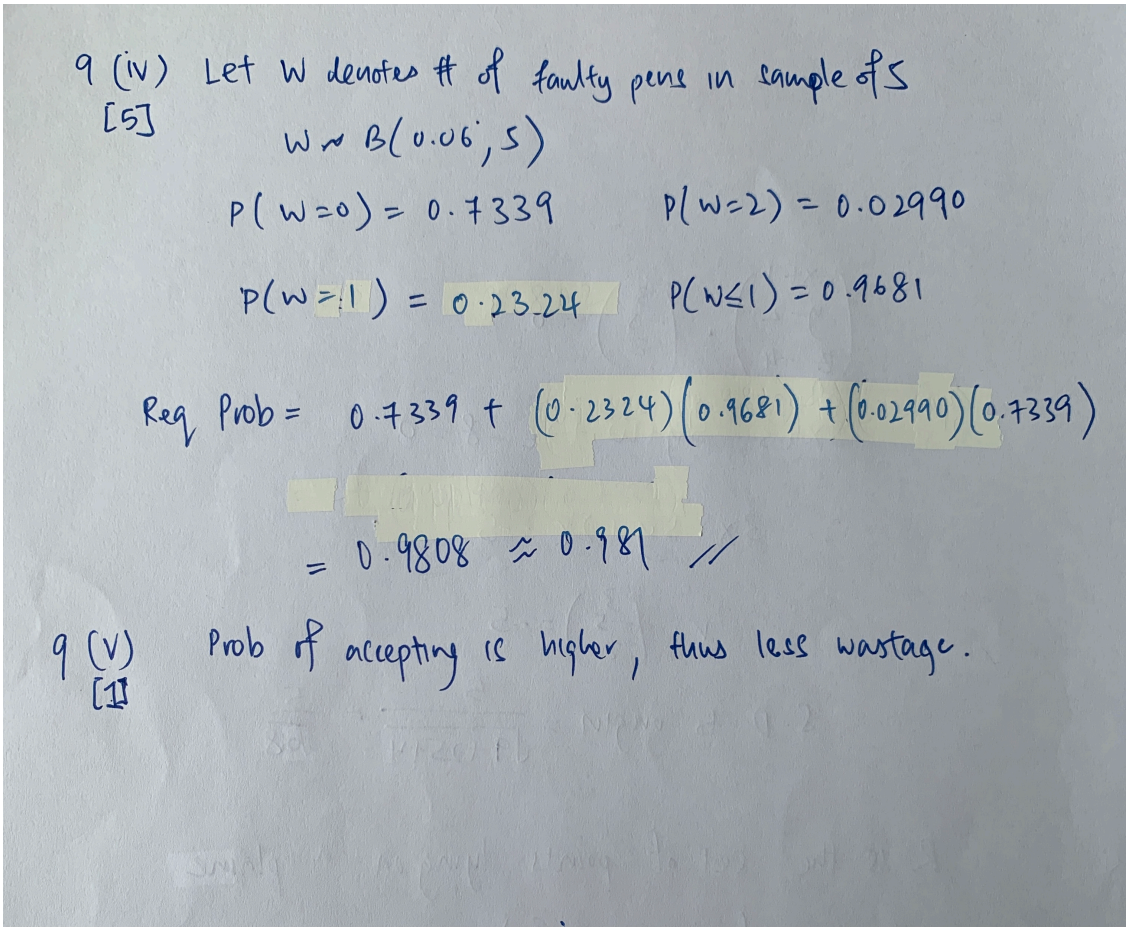

9 (iv) [5] Let W denotes # of faulty pens in sample of 5

$W \sim B(0.06, 5)$

$P(W=0) = 0.7339$ $\quad P(W=2) = 0.02990$

$P(W=1) = 0.2324$ $\quad P(W \leq 1) = 0.9681$

Req Prob $= 0.7339 + (0.2324)(0.9681) + (0.02990)(0.7339)$

$= 0.9808 \approx 0.981$ //

9 (v) [1] Prob of accepting is higher, thus less wastage.

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AO studies
where transFormAtion begins

10 (i) [4] Let $\bar{X}$ be sample mean of the amt of carbon in each bar

$H_0: \mu = 1.5$

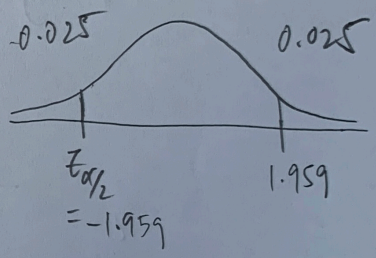
$H_1: \mu \neq 1.5$

Under H_0,

Test statistic : $z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

$= \dfrac{\bar{X} - 1.5}{0.09/\sqrt{15}}$

$= \dfrac{\bar{X} - 1.5}{0.023237}$



Critical region

$z < -1.959$ $\quad$ $z > 1.959$

$\bar{X} - 1.5 < -0.0455$ $\quad$ $\bar{X} - 1.5 > 0.0455$

$\bar{X} < 1.454$ $\quad$ $\bar{X} > 1.545$

$\therefore 0 < \bar{X} < 1.454$, $1.545 < \bar{X} < 100$ for H_0 to be rejected at 5% level of sig

(ii) [2] Since is new line of flat bars, they may not follow normal distribution.

$n = 40$, n is large, by CLT, the sample mean weight of flat bars follow normal dist.

(iii) [2]

$\bar{x} = 0.254$

$s^2 = \frac{1}{39}\left(2.586342 - \frac{(10.16)^2}{40}\right)$

$= 0.0001462$

(iv) [5]

$H_0: \mu = 0.25$

$H_1: \mu > 0.25$

Under H_0, by CLT $\because n$ is large $\bar{x} \sim N\left(0.25, \frac{s^2}{40}\right)$ approx

Test statistic: $z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = 2.0922$

From G.C. p-value $= 0.0182 < 0.025$

Hence, reject H_0 at 2.5% level of sig and conclude that there is sufficient evidence that the mean amount of carbon is more than 0.25%.