2020 GCE O'Level

Additional Mathematics Paper 1 (4047/01)

Suggested Answers

studies
where transFormAtion begins

Q1

Sum of roots : $\frac{5}{2} = \alpha + \beta$

product of roots : $4 = \alpha\beta$

New roots

Sum : $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\frac{25}{4} - 2(4)}{4}$

$= -\frac{7}{16}$

product : $1 = \frac{16}{16}$

$\therefore$ Quad Eq^n with roots $\frac{\alpha}{\beta}$ & $\frac{\beta}{\alpha}$ is $16x^2 + 7x + 16 = 0$ #

$= \frac{9}{50^2} \times 3^{\frac{3}{2}} \times \frac{2}{5}$

$= 3^2 . 5^{-4} . 2^{-2} . 3^{\frac{3}{2}} . 2 . 5^{-1}$

$= 3^{7/2} . 2^{-1} . 5^{-5}$

$\therefore \ a = -1 \ , \ b = \frac{7}{2} \ , \ c = -5$ #

$1\,000\,000 \, (1.07)^6$

$= 1{,}500{,}730.352$

$\approx 1{,}500{,}000$ (2 s.f.) #

$$\frac{4x^2-7x+9}{2x^2-x-3} = 2 + \frac{15-5x}{2x^2-x-3}$$

$$\begin{array}{r} 2 \\ 2x^2-x-3 \overline{)\,4x^2-7x+9} \\ -\,(4x^2-2x-6) \\ \hline 0 \;-5x+15 \end{array}$$

consider $\dfrac{15-5x}{2x^2-x-3}$

$$\frac{15-5x}{2x^2-x-3} = \frac{15-5x}{(2x-3)(x+1)} \equiv \frac{A}{2x-3} + \frac{B}{x+1}$$

sub $x = \frac{3}{2}$ $\quad A = \dfrac{15-5\left(\frac{3}{2}\right)}{\frac{3}{2}+1} = 3$

sub $x = -1$ $\quad B = \dfrac{15-5(-1)}{2(-1)-3} = -4$

$$\therefore \frac{4x^2-7x+9}{2x^2-x-3} = 2 + \frac{3}{2x-3} - \frac{4}{x+1}$$

Q4

$$\frac{dy}{dx} = \frac{(2)(x^2+4) - (2x)(2x-3)}{(2x-3)^2}$$

For y to be increasing, $\frac{dy}{dx} > 0$

$\because (2x-3)^2 > 0$ for all x

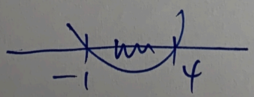
then $2(x^2+4) - (2x)(2x-3) > 0$

$x^2 + 4 - 2x^2 + 3x > 0$

$-x^2 + 3x + 4 > 0$

$x^2 - 3x - 4 < 0$

$(x-4)(x+1) < 0$

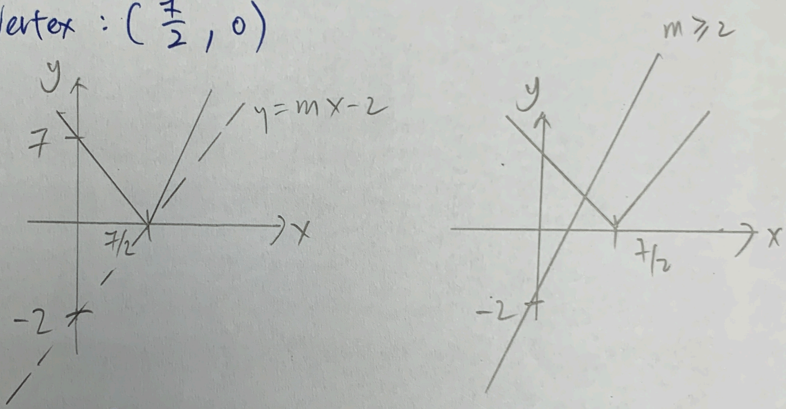


$\therefore -1 < x < 4$

#

Q5

Vertex : $\left(\frac{7}{2}, 0\right)$



Gradient : $\frac{2}{7/2} = \frac{4}{7}$

- when $m = \frac{4}{7}$, the line $y = mx - 2$ touch the graph of $y = |7 - 2x|$ at 1 point (vertex)
- when $m < \frac{4}{7}$, the line $y = mx - 2$ does not intersect the graph.
- when $m \geq 2$, the line $y = mx - 2$ cuts the graph at 1 point.

Hence, when $\frac{4}{7} < m < 2$, $y = mx - 2$ cuts the graph at 2 distinct points.

Q6

$y = x^2 + 4x^{-2}$

$\frac{dy}{dx} = 2x - 8x^{-3}$

$\frac{dy}{dx} = 0 \qquad 2x = \frac{8}{x^3}$

$x^4 = 4$

$x = \pm\sqrt{2}$

when $x = -\sqrt{2}$, $y = 4$, when $x = \sqrt{2}$, $y = 4$

$\frac{d^2y}{dx^2} = 2 + 24x^{-4}$

$\left.\frac{d^2y}{dx^2}\right|_{x=-\sqrt{2}} = 2 + \frac{24}{(-\sqrt{2})^4} > 0$

$\therefore$ $(-\sqrt{2}, 4)$ is a minimum point.

$\left.\frac{d^2y}{dx^2}\right|_{x=\sqrt{2}} = 2 + \frac{24}{(\sqrt{2})^4} > 0$

$\therefore$ $(\sqrt{2}, 4)$ is a minimum pt.

Q7

$$3\cos A = \frac{1}{\cos A} - \frac{5\sin A}{\cos A}$$

$$3\cos^2 A = 1 - 5\sin A$$

$$3(1-\sin^2 A) = 1 - 5\sin A$$

$$-3\sin^2 A + 5\sin A + 2 = 0$$

$\sin A = 2$ (rej) or $\sin A = -\frac{1}{3}$

$\alpha = 19.5^\circ$

$\therefore A = 199.5^\circ, 340.5^\circ$

Q8

let $f(x) = x^3 + ax$

By remainder theorem

$f(2) = 2^3 + 2a$

$= 8 + 2a$

$f(-1) = -1 - a$

$\Rightarrow 8 + 2a = -1 - a$

$3a = -9$

$a = -3$ //

let $f(x) = x^3 - 2x^2 - 4x + 3$

$f(3) = 3^3 - 2(3^2) - 4(3) + 3$

$= 0$

$\therefore$ By factor theorem, $(x-3)$ is a factor

$x^3 - 2x^2 - 4x + 3 \equiv (x-3)(x^2 + ax - 1)$

compare coefficient of x^2: $-2 = a - 3$

$a = 1$

let $x^2 + x - 1 = 0$

$x: \frac{-1 \pm \sqrt{1-4(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$\therefore x = 3, \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}$

Q9

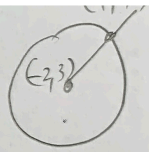
$$x^2+4x+y^2-6y=12$$

$$(x+2)^2-4+(y-3)^2-9=12$$

$$(x+2)^2+(y-3)^2=5^2$$

$\therefore$ centre $(-2,3)$ radius : 5 #

Gradient of normal: $\dfrac{7-3}{1-(-2)}=\dfrac{4}{3}$



$\therefore$ Gradient of tangent $=-\dfrac{3}{4}$

Eq^{n} : $y-7=-\dfrac{3}{4}(x-1)$

$$y=-\frac{3}{4}x+\frac{3}{4}+7$$

$$=-\frac{3}{4}x+\frac{31}{4}$$ #

$-\frac{\sqrt{3}}{2} = \cos 2x$

$\alpha = \frac{\pi}{6}$

$2x = \frac{5}{6}\pi, \frac{7\pi}{6}$

$x = \frac{5}{12}\pi, \frac{7\pi}{12}$

$\therefore$ x-coordinate of A is $\frac{5\pi}{12}$

x-coordinate of B is $\frac{7\pi}{12}$

$$\left|\int_{\frac{5\pi}{12}}^{\frac{7\pi}{12}} \cos 2x \, dx\right| = \left|\left[\frac{\sin 2x}{2}\right]_{5\pi/12}^{7\pi/12}\right|$$

$$= \left|\frac{1}{2}\sin\frac{7\pi}{6} - \frac{1}{2}\sin\frac{5\pi}{12}\right|$$

$$= \left|-\frac{1}{4} - \frac{1}{4}\right|$$

$$= \frac{1}{2}$$

$$\text{Req Area} = \frac{1}{2} - \left(\frac{2\pi}{12}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2} - \frac{\sqrt{3}\pi}{12} \text{ units}^2$$

$t=0 \quad V_A = 15(0+e^0) = 15 \text{ m/s}$

$\therefore V_B = 30 \text{ m/s}$

(ii) Find the distance between A and B. [6]

$\because e^{kt} > 0$ for all real values of k & t and $t \geq 0$

then $V \geq 0$ for the journey from A to B

$t=10, \; V=30$

$2 = \frac{1}{2} + e^{10k}$

$\frac{3}{2} = e^{10k}$

$k = \frac{1}{10}\ln\frac{3}{2}$

$D = \int_0^{10} v \, dt$

$= \int_0^{10} \frac{3}{4}t + 15e^{\frac{1}{10}\ln\frac{3}{2}t} \, dt$

$= \left[\frac{3}{8}t^2 + \frac{15}{\frac{1}{10}\ln\frac{3}{2}} e^{\frac{1}{10}\ln\frac{3}{2}t}\right]_0^{10}$

$= \left(\frac{300}{8} + 554.917\right) - (369.945)$

$= 222.47$

$\approx 222 \text{ m}$ (3sf)

$$a = \frac{dv}{dt} = \frac{3}{4} + 15\left(\frac{1}{10}\ln\frac{3}{2}\, e^{\frac{1}{10}\ln\frac{3}{2}t}\right)$$

when $t = 2$

$$a = \frac{3}{4} + \frac{15}{10}\ln\frac{3}{2}\, e^{\frac{1}{5}\ln\frac{3}{2}}$$

$$= 1.4095$$

$$\approx 1.41 \text{ m/s}^2$$

$$\text{RHS}: \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$= \frac{2\sin\alpha\cos\beta}{2\cos\alpha\cos\beta}$$

$$= \tan\alpha = \text{LHS} \ \#$$

Since $\sin(A-B)$ is negative, then $A-B<0$

Hence $A<B$

(iii) Find the value of $\cos(A-B)$. [2]

$$\cos(A-B) = \frac{4}{5} \quad (4^{th} \text{ Quad})$$

$\#$

$$\sin(A+B) = \frac{24}{7} \times \frac{7}{25} = \frac{24}{25} \quad //$$

(v) Using the identity from part (i), show that $\tan A = \frac{1}{3}$. [2]

$$\tan A = \frac{\frac{24}{25} + \left(-\frac{3}{5}\right)}{\frac{7}{25} + \frac{4}{5}} = \frac{\frac{9}{25}}{\frac{27}{25}} = \frac{1}{3} \quad \#$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{24}{7}$$

$$\Rightarrow \frac{\frac{1}{3} + \tan B}{1 - \frac{1}{3}\tan B} = \frac{24}{7}$$

$$\frac{7}{3} + 7\tan B = 24 - 8\tan B$$

$$15\tan B = \frac{65}{3}$$

$$\tan B = \frac{13}{9} \quad \#$$