2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

1 You are given that $I = \int x(1-x)^{\frac{1}{2}}\,dx$.

(i) Use integration by parts to find an expression for I. [2]

(i) $u = x$ $\quad \frac{dv}{dx} = \sqrt{1-x}$

$\frac{du}{dx} = 1$ $\quad v = -\frac{2}{3}(1-x)^{\frac{3}{2}}$

$$I = \int x(1-x)^{\frac{1}{2}}\,dx = \left[-\frac{2x}{3}(1-x)^{3/2}\right] + \int \frac{2}{3}(1-x)^{3/2}\,dx$$

$$= -\frac{2x}{3}(1-x)^{3/2} - \frac{2}{3}\left(\frac{2}{5}\right)(1-x)^{5/2} + C$$

$$= -\frac{2x}{3}(1-x)^{3/2} - \frac{4}{15}(1-x)^{5/2} + C \quad \#$$

(ii) Use the substitution $u^2 = 1 - x$ to find another expression for I. [2]

(ii) $u^2 = 1 - x$

$2u\left(\frac{du}{dx}\right) = -1$

$\frac{du}{dx} = -\frac{1}{2\sqrt{1-x}}$

$$I = \int x(1-x)^{\frac{1}{2}}\,dx$$

$$= \int (1-u^2)(1-x)^{\frac{1}{2}}\left(-2\sqrt{1-x}\right)du$$

$$= -2\int (1-u^2)(u^2)\,du$$

$$= -2\left[\frac{1}{3}u^3 - \frac{1}{5}u^5\right] + C_1$$

$$= -\frac{2}{3}(1-x)^{\frac{3}{2}} + \frac{2}{5}(1-x)^{\frac{5}{2}} + C_1$$

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

(iii) Show algebraically that your answers to parts **(i)** and **(ii)** differ by a constant. **[2]**

1 (iii)

$$-\frac{2}{3}(1-x)^{3/2}+\frac{2}{5}(1-x)^{5/2}+C_1-\left(-\frac{2x}{3}(1-x)^{3/2}-\frac{4}{15}(1-x)^{5/2}+C\right)$$

$$=\left(-\frac{2}{3}+\frac{2}{3}x\right)(1-x)^{3/2}+\frac{2}{3}(1-x)^{5/2}+C_1-C$$

$$=-\frac{2}{3}(1-x)(1-x)^{3/2}+\frac{2}{3}(1-x)^{5/2}+C_1-C$$

$$=-\frac{2}{3}(1-x)^{5/2}+\frac{2}{3}(1-x)^{5/2}+C_1-C$$

$$=C_1-C \quad = \text{constant}.$$

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

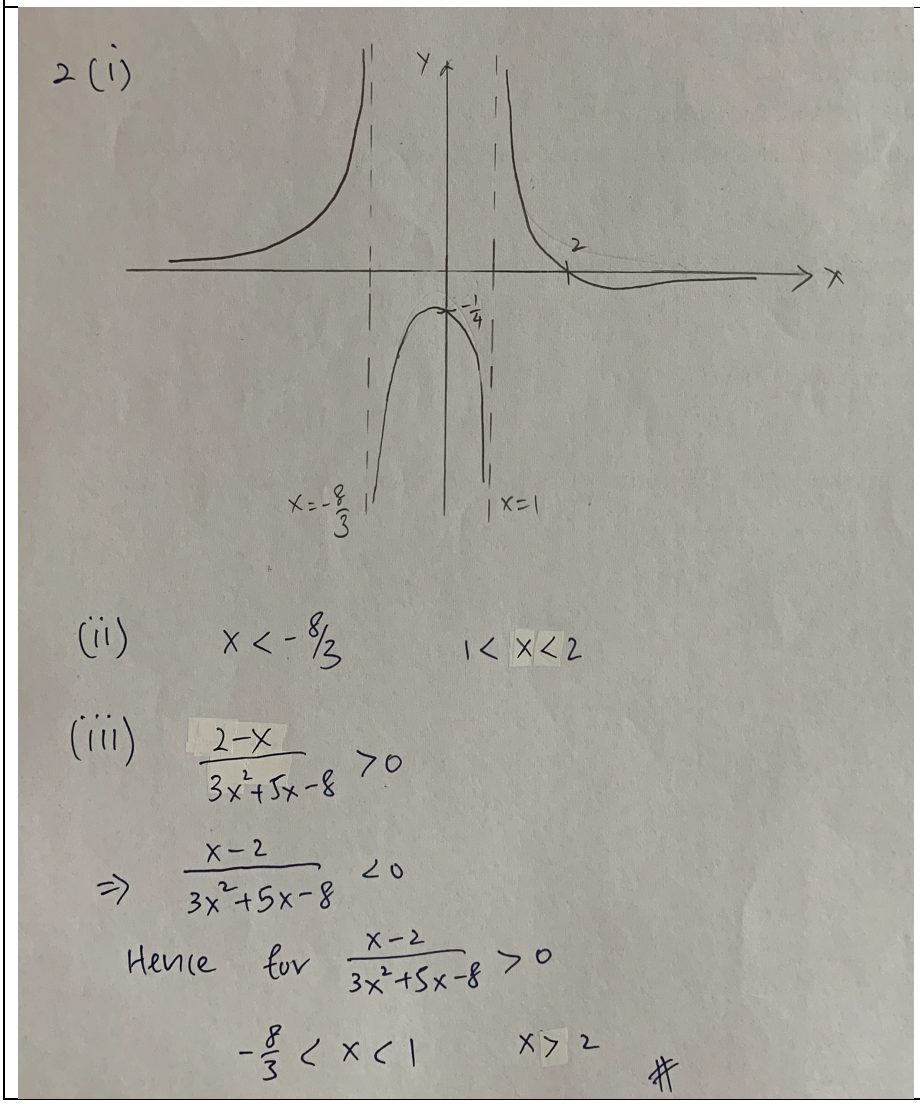
Suggested Answers

AO studies
where transFormAtion begins

2 (i) Sketch the graph of $y = \dfrac{2-x}{3x^2+5x-8}$. Give the equations of the asymptotes and the coordinates of the point(s) where the curve crosses either axis. [4]

(ii) Solve the inequality $\dfrac{2-x}{3x^2+5x-8} > 0$. [1]

(iii) Hence solve the inequality $\dfrac{x-2}{3x^2+5x-8} > 0$. [1]



3 A solid cylinder has radius r cm, height h cm and total surface area 900 cm^2. Find the exact value of the maximum possible volume of the cylinder. Find also the ratio $r : h$ that gives this maximum volume. [7]

(3)

$2\pi rh + 2\pi r^2 = 900$

$rh + r^2 = \frac{900}{2\pi}$

$h = \frac{\frac{450}{\pi} - r^2}{r} = \frac{450 - \pi r^2}{\pi r}$

$V = \pi r^2 h$

$= \pi r^2 \left(\frac{450 - \pi r^2}{\pi r}\right)$

$= 450r - \pi r^3$

$\frac{dV}{dr} = 450 - 3\pi r^2$ $\qquad \frac{d^2V}{dr^2} = -6\pi r$

$\frac{dV}{dr} = 0 \Rightarrow 150 = \pi r^2$ $\qquad \left.\frac{d^2V}{dr^2}\right|_{r=\sqrt{\frac{150}{\pi}}} < 0$ $\therefore V$ is max.

$r = \sqrt{\frac{150}{\pi}}$

$V_{max} = 450\sqrt{\frac{150}{\pi}} - 150\sqrt{\frac{150}{\pi}}$

$= 300\sqrt{\frac{150}{\pi}}$

$= \frac{300(5)\sqrt{6\pi}}{\pi}$

$= \frac{1500}{\pi}\sqrt{6\pi}$ cm^3 //

$h = \frac{450 - \pi\left(\frac{150}{\pi}\right)}{\pi\sqrt{\frac{150}{\pi}}} = \frac{300}{\sqrt{150\pi}}$

$= \frac{2\sqrt{150}}{\sqrt{\pi}} = 2\sqrt{\frac{150}{\pi}}$

$\therefore r : h$

$1 : 2$ #

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

4 (i) Given that $f(x) = \sec 2x$, find $f'(x)$ and $f''(x)$. Hence, or otherwise, find the Maclaurin series for $f(x)$, up to and including the term in x^2. [5]

(ii) Use your series from part (i) to estimate $\int_0^{0.02} \sec 2x \, dx$, correct to 5 decimal places. [2]

4 (i) $f(x) = \sec 2x$

$f'(x) = 2\sec 2x \tan 2x$

$f''(x) = 2\left(\sec 2x \tan^2 2x (2) + \sec^3 2x (2)\right)$

$= 4\left(\sec 2x \tan^2 2x + \sec^3 2x\right)$

$f(0) = 1$

$f'(0) = 0$

$f''(0) = 4$

$\therefore f(x) = 1 + \frac{x^2}{2}(4) + \ldots$

$= 1 + 2x^2 + \ldots$

(ii) $\int_0^{0.02} \sec 2x \, dx \approx \int_0^{0.02} 1 + 2x^2 \, dx$

$= \left[x + \frac{2}{3}x^3\right]_0^{0.02}$

$= 0.02000533$

≈ 0.02001 #

(iii) Use your calculator to find $\int_0^{0.02} \sec 2x \, dx$, correct to 5 decimal places. [1]

(iv) Comparing your answers to parts **(ii)** and **(iii)**, and with reference to the value of x, comment on the accuracy of your approximations. [2]

(v) Explain why a Maclaurin series for $g(x) = \operatorname{cosec} 2x$ cannot be found. [1]

(iii) From G.C. $\int_0^{0.02} \sec 2x \, dx = 0.02001$

(iv) Both (ii) & (iii) are same up to five decimal places. $x = 0.02$ is close to 0. Since Maclaurin series is centre at $x=0$, the approximation obtained in (ii) is accurate.

(v) $g(x) = \operatorname{cosec} 2x$ is undefined at $x=0$. Hence, Maclaurin series cannot be used.

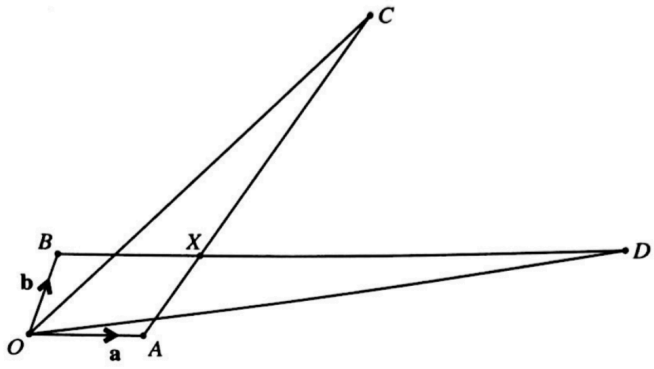
2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

5



With reference to the origin O, the points A, B, C and D are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = 2\mathbf{a} + 4\mathbf{b}$ and $\overrightarrow{OD} = \mathbf{b} + 5\mathbf{a}$. The lines BD and AC cross at X (see diagram).

(i) Express $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [4]

(5) (i) $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$
$= -\underset{\sim}{b} + \underset{\sim}{b} + 5\underset{\sim}{a}$
$= 5\underset{\sim}{a}$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$
$= -\underset{\sim}{a} + 2\underset{\sim}{a} + 4\underset{\sim}{b}$
$= \underset{\sim}{a} + 4\underset{\sim}{b}$

$l_{BD} = \underset{\sim}{b} + \lambda(5\underset{\sim}{a})$ where $\lambda \in \mathbb{R}$

$l_{AC} = \underset{\sim}{a} + \alpha(4\underset{\sim}{b} + \underset{\sim}{a})$ where $\alpha \in \mathbb{R}$

$\underset{\sim}{b} + 5\lambda\underset{\sim}{a} = \underset{\sim}{a}(1+\alpha) + 4\alpha\underset{\sim}{b}$

$4\alpha = 1$ $\quad 5\lambda = 1 + \alpha$

$\alpha = 1/4$ $\quad \lambda = \frac{1}{4}$

$\therefore \overrightarrow{OX} = \underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a}$ #

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

The point Y lies on CD and is such that the points O, X and Y are collinear.

(ii) Express $\overrightarrow{OY}$ in terms of **a** and **b** and find the ratio $OX : OY$. **[6]**

(ii) $\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$

$= -2\underset{\sim}{a} - 4\underset{\sim}{b} + \underset{\sim}{b} + 5\underset{\sim}{a}$

$= 3\underset{\sim}{a} - 3\underset{\sim}{b}$

$l_{OX} = \beta(\underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a})$ where $\beta \in \mathbb{R}$

$l_{CD} = 2\underset{\sim}{a} + 4\underset{\sim}{b} + \phi(3\underset{\sim}{a} - 3\underset{\sim}{b})$ where $\phi \in \mathbb{R}$

$2\underset{\sim}{a} + 4\underset{\sim}{b} + \phi(3\underset{\sim}{a} - 3\underset{\sim}{b}) = \beta(\underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a})$

$2 + 3\phi = \frac{5}{4}\beta$
$4 - 3\phi = \beta$

$\phi = \frac{4}{9}$, $\beta = \frac{8}{3}$ from G.C.

$\therefore \overrightarrow{OY} = \frac{8}{3}(\underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a})$

$= \frac{8}{3}\overrightarrow{OX}$

$\therefore OX : OY$

$3 : 8$

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

Section B: Probability and Statistics [60 marks]

6 In a certain country there are 100 professional football clubs, arranged in 4 divisions. There are 22 clubs in Division One, 24 in Division Two, 26 in Division Three and 28 in Division Four.

(i) Alice wishes to find out about approaches to training by clubs in Division One, so she sends a questionnaire to the 22 clubs in Division One. Explain whether these 22 clubs form a sample or a population. [1]

(ii) Dilip wishes to investigate the facilities for supporters at the football clubs, but does not want to obtain the detailed information necessary from all 100 clubs. Explain how he should carry out his investigation, and why he should do the investigation in this way. [2]

(iii) Find the number of different possible samples of 20 football clubs, with 5 clubs chosen from each division. [3]

(6)(i) Sample. They are drawn from 100 football clubs.

(ii) He can randomly select 2 clubs from each division to investigate the facilities. This will prevent unbiasness and also ensure his investigation includes clubs from all division.

(iii) $^{22}C_5 \times {}^{24}C_5 \times {}^{26}C_5 \times {}^{28}C_5 = 7.24 \times 10^{18}$ ways

7 A company produces drinking mugs. It is known that, on average, 8% of the mugs are faulty. Each day the quality manager collects 50 of the mugs at random and checks them; the number of faulty mugs found is the random variable F.

(i) State, in the context of the question, two assumptions needed to model F by a binomial distribution. [2]

(7) (i) 1. The probability of each randomly chosen mug is faulty is constant throughout

2. The mugs are either faulty or not faulty.

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

You are now given that F can be modelled by a binomial distribution.

(ii) Find the probability that, on a randomly chosen day, at least 7 faulty mugs are found. [2]

(iii) The number of faulty mugs produced each day is independent of other days. Find the probability that, in a randomly chosen working week of 5 days, at least 7 faulty mugs are found on no more than 2 days. [2]

The company also makes saucers. The number of faulty saucers also follows a binomial distribution. The probability that a saucer is faulty is p. Faults on saucers are independent of faults on mugs.

(iv) Write down an expression in terms of p for the probability that, in a random sample of 10 saucers, exactly 2 are faulty. [1]

(ii) $F \sim B(50, 0.08)$

$P(F \geq 7) = 0.102$ #

(iii) Let X denotes # of day with more than 7 faulty mugs

$X \sim B(5, 0.10187)$

$P(X \leq 2) = 0.991$ #

(iv) Let S denotes # of faulty saucers.

$S \sim B(10, p)$

$P(S=2) = {}^{10}C_2\, p^2(1-p)^8$

$= 45p^2(1-p)^8$ #

The mugs and saucers are sold in sets of 2 randomly chosen mugs and 2 randomly chosen saucers. The probability that a set contains at most 1 faulty item is 0.97.

(v) Write down an equation satisfied by p. Hence find the value of p. [4]

(v) Prob (mug faulty, saucer not faulty) + Prob (saucer faulty, mug not faulty)
+ Prob (none faulty)

$= [{}^2C_1(0.08)(0.92)][(1-p)^2] + [{}^2C_1(p)(1-p)][0.92^2] + [0.92^2](1-p)^2$

$\Rightarrow 0.1472(1-p)^2 + 1.6928p(1-p) + 0.8464(1-p)^2 = 0.97$

From G.C. $p = 0.06889$

≈ 0.0689 #

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

8 Gerri collects characters given away in packets of breakfast cereal. There are four different characters: Horse, Rider, Dog and Bird. Each character is made in four different colours: Orange, Yellow, Green and White. Gerri has collected 56 items; the numbers of each character and colour are shown in the table.

	Orange	Yellow	Green	White
Horse	1	1	3	4
Rider	1	1	7	5
Dog	3	7	1	6
Bird	4	5	6	1

(i) Gerri puts all the items in a bag and chooses one item at random.

(a) Find the probability that this item is either a Horse or a Rider. [1]

(b) Find the probability that this item is either a Dog or a Bird but the item is not White. [1]

(ii) Gerri now puts the item back in the bag and chooses two items at random.

(a) Find the probability that both of the items are Horses, but neither of the items is Orange. [1]

(b) Find the probability that Gerri's two items include exactly one Dog and exactly one item that is Yellow. [3]

(iii) Gerri has two favourites among the 16 possible colour/character combinations. The probability of choosing these two at random from the 56 items is $\frac{1}{77}$. Write down all the possibilities for Gerri's two favourite colour/character combinations. [3]

(8) i a) Prob : $\frac{9}{56} + \frac{14}{56} = \frac{23}{56}$ #

i b) prob : $\frac{3+7+1+4+5+6}{56} = \frac{13}{28}$ #

(ii) a) Prob : $\frac{8}{56} \times \frac{7}{55} = \frac{1}{55}$

b) Prob : $\left(\frac{3+1+6}{56} \times \frac{7}{55}\right)2 + \left(\frac{7}{56} \times \frac{32}{55}\right)2 = \frac{21}{110}$ #

(iii) $\frac{56 \times 55}{(77)2} = 20$

∴ possible combination : 5 x 4 (orange bird, yellow bird)
5 x 4 (white Horse, white rider)
5 x 4 (orange bird, white rider)
5 x 4 (yellow bird, white horse) #

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

9 A company produces resistors rated at 750 ohms for use in electronic circuits. The production manager wishes to test whether the mean resistance of these resistors is in fact 750 ohms. He knows that the resistances are normally distributed with variance 100 ohms^2.

(i) Explain whether the manager should carry out a 1-tail test or a 2-tail test. State hypotheses for the test, defining any symbols you use. [2]

The production manager takes a random sample of 8 of these resistors. He finds that the resistances, in ohms, are as follows.

742 771 768 738 769 752 742 766

(ii) Find the mean of the sample of 8 resistors. Carry out the test, at the 5% level of significance, for the production manager. Give your conclusion in context. [5]

The company also produces resistors rated at 1250 ohms. Nothing is known about the distribution of the resistances of these resistors.

(iii) Describe how, and why, a test of the mean resistance of the 1250 ohms resistors would need to differ from that for the 750 ohms resistors. [2]

9(i) 2-tail test since the resistance can be more or less than 750 Ω

$H_0: \mu = 750$

$H_1: \mu \neq 750$ where μ is the mean resistance

H_0 is null hypothesis

H_1 is alternative hypothesis

9(ii) mean = 756

Under H_0: $z = \dfrac{\bar{x} - 750}{\sqrt{\frac{100}{8}}}$

From G.C. p-value 0.08968 > 0.05

∴ Do not rej H_0.

Insufficient evidence to show mean resistance is not 750 ohms.

(iii) Since distribution is unknown, the sample size must be at least 30 in order to approximate the mean distribution by Central limit Theorem.

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

10 Abi and Bhani find the fuel consumption for a car driven at different constant speeds. The table shows the fuel consumption, y kilometres per litre, for different constant speeds, x kilometres per hour.

x	40	45	50	55	60
y	22	20	18	17	16

(i) Abi decides to model the data using the line $y = 35 - \frac{1}{3}x$.

(a) On the grid opposite

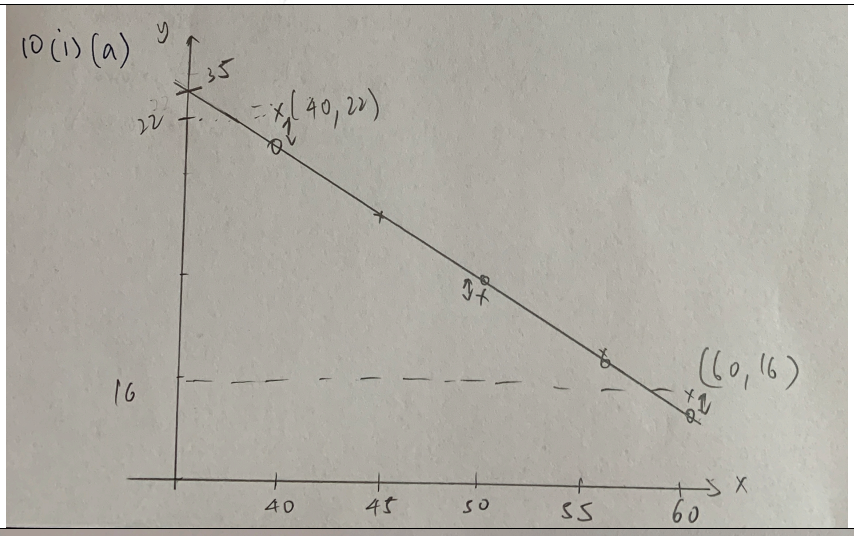
- draw a scatter diagram of the data,
- draw the line $y = 35 - \frac{1}{3}x$. [2]

(b) For a line of best fit $y = f(x)$, the residual for a point (a, b) plotted on the scatter diagram is the vertical distance between $(a, f(a))$ and (a, b). Mark the residual for each point on your diagram. [1]

(c) Calculate the sum of the squares of the residuals for Abi's line. [1]

(d) Explain why, in general, the sum of the squares of the residuals rather than the sum of the residuals is used. [1]

10(i)(a)



10(i)(c) $y = -\frac{1}{3}x + 33.6$ from G.C.

From G.C. Sum of sq of residuals : $\frac{4}{3}$

(d) Sum of residuals may be negative and thus, summing them up may give an erroneous results.

10 [Continued]

Bhani models the same data using a straight line passing through the points (40, 22) and (55, 17). The sum of the squares of the residuals for Bhani's line is 1.

(ii) State, with a reason, which of the two models, Abi's or Bhani's, gives a better fit. [1]

(iii) State the coordinates of the point that the least squares regression line must pass through. [1]

(iv) Use your calculator to find the equation of the least squares regression line of y on x. State the value of the product moment correlation coefficient. [3]

(v) Use the equation of the regression line to estimate the fuel consumption when the speed is 30 kilometres per hour. Explain whether you would expect this value to be reliable. [2]

(vi) Cerie performs a similar experiment on a different car. She finds that the sum of the squares of the residuals for her line is 0. What can you deduce about the data points in Cerie's experiment? [1]

10 (ii) Bhani's model. Sum of residual is smaller.

(iii) Sample mean at (50, 18.6)

(iv) $y = -0.3x + 33.6$

$r = -0.985$

(v) $y = -0.3(30) + 33.6$

$= 24.6$ km/l

Not reliable. Out of data range.

(vi) Since sum of the squares of residual is 0, then all the data points lies on her line

2019 GCE A'Level

H2 Mathematics Paper 2 (9758/02)

Suggested Answers

AO studies
where transFormAtion begins

11 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

Arif is making models of hydrocarbon molecules. Hydrocarbons are chemical compounds made from carbon atoms and hydrogen atoms. Arif has a bag containing a large number of white balls to represent the carbon atoms, and a bag containing a large number of black balls to represent the hydrogen atoms. The masses of the white balls have the distribution $N(110, 4^2)$ and the masses of the black balls have the distribution $N(55, 2^2)$. The units for mass are grams.

(i) Find the probability that the total mass of 4 randomly chosen white balls is more than 425 grams. [2]

(ii) Find the probability that the total mass of a randomly chosen white ball and a randomly chosen black ball is between 161 and 175 grams. [2]

11 (i) Let W be r.v. denoting mass of white balls
Let B be r.v denoting mass of black balls

$W \sim N(110, 4^2)$ $B \sim N(55, 2^2)$

$W_1 + W_2 + W_3 + W_4 \sim N(440, 64)$

$P(W_1 + W_2 + W_3 + W_4 > 425) = 0.970$ #

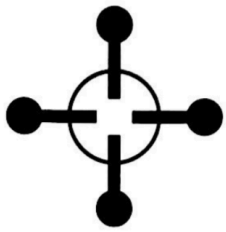
(ii) $W + B \sim N(165, 20)$

$P(161 < W + B < 175) = 0.802$ #

(iii) The probability that 2 randomly chosen white balls and 3 randomly chosen black balls have total mass less than M grams is 0.271. Find the value of M. [4]

11 [Continued]

Arif also has a bag containing a large number of connecting rods to fix the balls together. The masses of the connecting rods, in grams, have the distribution $N(20, 0.9^2)$. In order to make models of methane (a hydrocarbon), Arif has to drill 1 hole in each black ball, and 4 holes in each white ball, for the connecting rods to fit in. This reduces the mass of each black ball by 10% and reduces the mass of each white ball by 30%.



A methane molecule consists of 1 carbon atom and 4 hydrogen atoms. Arif makes a model of a methane molecule using 4 black balls, 1 white ball and 4 connecting rods (see diagram). The balls and connecting rods are all chosen at random.

(iv) Find the probability that the mass of Arif's model is more than 350 grams. [4]

(iii) $W_1 + W_2 + B_1 + B_2 + B_3 \sim N(385, 44)$

$P(W_1 + \ldots + B_3 < m) = 0.271$

$m = 380.95$

≈ 381 grams

(iv) $B_1 + B_2 + B_3 + B_4 \sim N(220, 16)$ Let R denote mass of rods

$0.9(B_1 + B_2 + B_3 + B_4) \sim N(198, 12.96)$ $R_1 + R_2 + R_3 + R_4 \sim N(80, 3.24)$

$0.7(W) \sim N(77, 7.84)$

$0.9(B_1 + \ldots + B_4) + 0.7W + (R_1 + R_2 + R_3 + R_4) \sim N(355, 24.04)$

$P([0.9(B_1 + \ldots + B_4) + 0.7W + (R_1 + \ldots + R_4)] > 350) = 0.846$ #