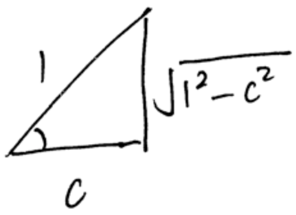
2019 GCE O'Level

Additional Mathematics Paper 1 (4047/01)

Suggested Answers

1 Given that θ is acute and $\cos\theta = c$, express, in terms of c,

(i) $\tan\theta$, [3]



$\therefore \tan\theta = \dfrac{\sqrt{1-c^2}}{c}$ #

(ii) $\operatorname{cosec}\theta$. [1]

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\sqrt{1-c^2}}$

2 Find the set of values of the constant k for which the curve $y = x^2 + (2k+1)x + 1$ lies entirely above the line $y = x$. [4]

$y = x^2 + (2k+1)x + 1$

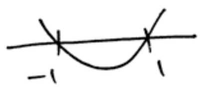
$y = x$

$\Rightarrow x^2 + (2k+1)x + 1 = x$

$x^2 + 2kx + 1 = 0$

$D: \quad 4k^2 - 4(1)(1) < 0$

$k^2 - 1 < 0$



$\Rightarrow -1 < k < 1$

Set of values : $\{k \in \mathbb{R} : -1 < k < 1\}$

3 Given that $y = Ae^{2x} + Be^{-x}$, and that $\frac{dy}{dx} + 4y = e^{2x} - e^{-x}$, find the value of each of the constants A and B. [4]

$$y = Ae^{2x} + Be^{-x}$$

$$\frac{dy}{dx} = 2Ae^{2x} - Be^{-x}$$

$$\Rightarrow \frac{dy}{dx} + 4y = 2Ae^{2x} - Be^{-x} + 4Ae^{2x} + 4Be^{-x}$$

$$= 6Ae^{2x} + 3Be^{-x}$$

$\therefore 6A = 1$ $\quad$ $3B = -1$

$A = 1/6$ $\quad$ $B = -1/3$ #

6

4 An ice cube of side x cm is melting in such a way that the total surface area, A cm^2, is decreasing at a constant rate of 48 cm^2/s. Assuming that the cube retains its shape, calculate the rate of change of x when $x = 10$. [4]

$$\frac{dA}{dt} = -48 \text{ cm}^2/\text{s}$$

$$A = 6x^2$$

$$\frac{dA}{dx} = 12x$$

$$\therefore \left.\frac{dA}{dx}\right|_{x=10} = 120$$

$$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}$$

$$-48 = 120\left(\frac{dx}{dt}\right)$$

$$\therefore \frac{dx}{dt} = -\frac{2}{5} \text{ cm}/\text{s}$$

5 (i) A manufacturer produces a disinfectant that destroys 21% of all known germs within one minute of use. If N is the number of germs present when the disinfectant is first used, and assuming germs continue to be destroyed at the same rate, explain why the number of germs expected to be alive after n minutes is given by $(0.79)^n N$. [2]

Since 21% of germs killed in 1 min

then $(0.79)N$ germs present after 1 min

Since germs continue to be destroyed at the same rate, then after 2mins, $0.79(0.79N)$ germs left.

Hence, after n mins, number of germs alive is $(0.79)^n N$.

(ii) The manufacturer decides to advertise by stating that the disinfectant destroys x% of all known germs within 20 minutes of use. Calculate, to 2 significant figures, the value of x. [2]

$$(0.79)^{20} N = (0.0089648)N$$

No. of germs destroyed: $N - 0.0089648N$

$$= 0.991035N$$

$$\approx 0.99N$$

Hence $x = 99\%$ #

(iii) Given that the number of germs expected to be alive after n minutes can be expressed as Ne^{kn}, find the value of the constant k. [2]

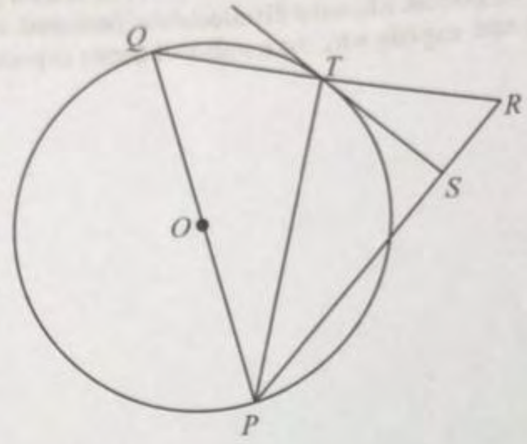
$$(0.79^n)N = Ne^{kn}$$

$$0.79^n = e^{kn}$$

$$n(\ln 0.79) = kn$$

$$k = -0.2357 \approx -0.236 \ \#$$

6



In the diagram, PQ is the diameter of a circle, centre O. Triangle PQR is an isosceles triangle with $PQ = PR$. The line QR intersects the circle at T. The tangent to the circle at T meets PR at S.

(i) Show that angle $TSR = 90°$. [5]

$\because$ QP is diameter

$\angle QTP = 90°$ ($\angle$ in semi-circle)

Hence T bisects QR $\Rightarrow \angle QPT = \angle TPS$

$\angle STP = \angle TQP$ (Alt. segment Theorem)

Using $\triangle QTP$: $\angle QTP + \angle TQP + \angle QPT = 180°$ (sum of $\angle$ in $\triangle$)

$\Rightarrow \angle QPT + \angle TQP = 180° - 90° = 90°$

Hence $\angle STP + \angle TPS = 90°$

Using $\triangle SPT$: $\angle STP + \angle TPS + \angle TSP = 180°$

$\angle TSP = 180° - 90°$

$= 90°$

$\because$ PSR is a straight line

$\angle TSR = 180° - \angle TSP$

$= 90°$ shown

\#

(ii) Explain why the circle passing through the points S, R and T has its centre at the midpoint of TR. [2]

Since circle passes through TSR and $\angle TSR = 90°$, by properties of circle, angle in semi-circle is $90°$. Hence, TR is the diameter of the circle. Then it is clear that the centre of circle lies on the midpoint of diameter TR. #

7 (i) Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2-\frac{x}{8}\right)^6$. [3]

$$\left(2-\frac{x}{8}\right)^6 = {}^6C_0(2)^6\left(-\frac{x}{8}\right)^0 + {}^6C_1(2)^5\left(-\frac{x}{8}\right)^1 + {}^6C_2(2^4)\left(-\frac{x}{8}\right)^2 + \dots$$

$$= 64 - 24x + \frac{15}{4}x^2 + \dots$$

(ii) In the expansion of $(4+kx+x^2)\left(2-\frac{x}{8}\right)^6$, the sum of the coefficients of x and x^2 is zero. Find the value of the constant k. [4]

$$(4+kx+x^2)\left(2-\frac{x}{8}\right)^6 = (4+kx+x^2)\left(64-24x+\frac{15}{4}x^2+\dots\right)$$

coefficient of x : $4(-24) + 64k = -96 + 64k$

coefficient of x^2 : $15 - 24k + 64 = 79 - 24k$

$$\Rightarrow -96 + 79 + 64k - 24k = 0$$

$$-17 + 40k = 0$$

$$k = \frac{17}{40}$$

#

8 The equation of a curve is $y = x + \dfrac{2x+5}{x-2}$.

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

$$y = x + \frac{2x+5}{x-2}$$

$$\frac{dy}{dx} = 1 + \frac{(2)(x-2)-(1)(2x+5)}{(x-2)^2}$$

$$= 1 + \frac{2x-4-2x-5}{(x-2)^2}$$

$$= 1 - \frac{9}{(x-2)^2}$$

$$= 1 - 9(x-2)^{-2}$$

$$\frac{d^2y}{dx^2} = 18(x-2)^{-3} = \frac{18}{(x-2)^3}$$

#

(ii) Find the x-coordinate of each of the stationary points of the curve. [3]

$$\frac{dy}{dx} = 0 \Rightarrow 1 = \frac{9}{(x-2)^2}$$

$$(x-2)^2 = 9$$

$x-2 = 3$ or $x-2 = -3$

$x = 5$ $\quad$ $x = -1$

#

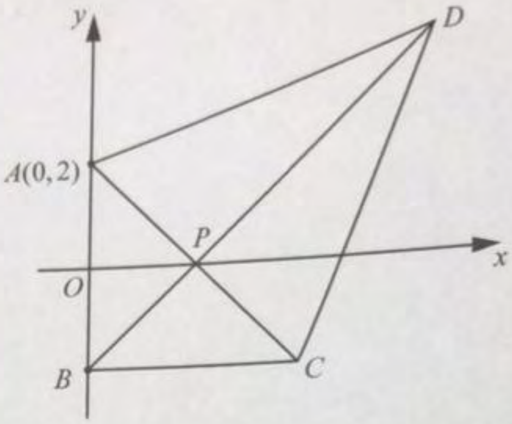
(iii) Find the nature of each stationary point. [2]

$\left.\frac{d^2y}{dx^2}\right|_{x=5} = \frac{18}{(5-2)^3} > 0 \quad \therefore$ minimum point

$\left.\frac{d^2y}{dx^2}\right|_{x=-1} = \frac{18}{(-1-2)^3} < 0 \quad \therefore$ maximum point.

#

9



The diagram shows a kite $ABCD$ in which $AB = BC$ and $AD = DC$. The points $A(0, 2)$ and B lie on the y-axis. The diagonals AC and BD intersect at the point P on the x-axis.
Given that the length of AB is 4 units,

(i) explain why BC is parallel to the x-axis, [2]

line $BD \perp$ line AC

$\Rightarrow \angle APB = 90^\circ$

$AB = 4$ units $\quad OA = 2$ units

then $OB = 2$ units $= OA$

so O is the mid point of AB.

then $\triangle APB$ is isosceles $\triangle$

$\angle PAB + \angle ABP = 180^\circ - 90^\circ$

$\Rightarrow \angle PAB = \angle ABP$
$= 45^\circ$

Given $AB = BC$, $\angle PCB = \angle PAB$
$= 45^\circ$

studies Signature Series
where transFormAtion begins

Suggested Answers

Using $\triangle ABC$ $\angle CAB + \angle ABC + \angle ACB$

$= \angle PAB + \angle ABC + \angle PCB$

$= 45^\circ + \angle ABC + 45^\circ$

$= 180^\circ$ (Sum of $\angle$ in $\triangle$)

$\therefore \angle ABC = 180^\circ - 45^\circ - 45^\circ = 90^\circ$

Hence BC is parallel to x-axis

(ii) find the coordinates of C. [1]

$\because BC = AB = 4$

Then $C(4,-2)$

Given further that the area of the kite is 28 units^2,

(iii) find the coordinates of D. [5]

$A(0,2)$ $B(0,-2)$ $C(4,-2)$ $D(x,y)$

$$\text{Area: } \frac{1}{2}\begin{vmatrix} 0 & 0 & 4 & x & 0 \\ 2 & -2 & -2 & y & 2 \end{vmatrix}$$

$$= \frac{1}{2}\left| 4y + 2x - (-2x - 8) \right|$$

$$= \frac{1}{2}\left| 4y + 4x + 8 \right| = 28$$

$$\left| y + x + 2 \right| = 14$$

Suggested Answers

$\text{Grad}_{AC} = \frac{-2-2}{4-0} = -1$

$\Rightarrow \text{Grad}_{BD} = 1$

$\therefore$ Eq^n of BD : $y = x - 2$

Sub in $|x - 2 + x + 2| = 14$

$|2x| = 14$

$x = 7$ or -7 (rej)

$\therefore y = 5$

coordinate $D(7, 5)$

10 (a) Find the values of x and y which satisfy the equations

$$3^{x+y} = \sqrt[3]{27},$$
$$\frac{4^y}{2^x} = \left(\frac{1}{2}\right)^{-3}.$$

[4]

$3^{x+y} = \sqrt[3]{27} = 3$

$\therefore\ x + y = 1$ — (1)

$\frac{4^y}{2^x} = \left(\frac{1}{2}\right)^{-3} = 8$

$2^{2y-x} = 8 = 2^3$

$2y - x = 3$ — (2)

(1) + (2) : $3y = 4$

$y = \frac{4}{3}$

$x = -\frac{1}{3}$ #

) A circular cylinder of volume $(3\sqrt{7}-6)\pi$ cm^3 has a height of $(2+\sqrt{7})$ cm and a radius of r cm. Without using a calculator, obtain an expression for r^2 in the form $(a+b\sqrt{7})$, where a and b are integers. [4]

Vol: $\pi r^2 h = (3\sqrt{7}-6)\pi$

$r^2(2+\sqrt{7}) = 3\sqrt{7}-6$

$r^2 = \dfrac{3\sqrt{7}-6}{2+\sqrt{7}} \times \dfrac{2-\sqrt{7}}{2-\sqrt{7}}$

$= \dfrac{6\sqrt{7}-21-12+6\sqrt{7}}{4-7}$

$= \dfrac{-33+12\sqrt{7}}{-3}$

$= 11-4\sqrt{7}$

$\therefore a=11, \ b=-4$ #

11 A dot on a computer screen moves in a straight line so that, t seconds after leaving a fixed point O, its displacement, s cm, from O is modelled by $s = t^3 - 6t^2 + 9t$.

(i) Find the values of t at which the dot is instantaneously at rest. [3]

$s = t^3 - 6t^2 + 9t$

$v = \frac{ds}{dt} = 3t^2 - 12t + 9$

$v = 0 \quad 3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3$ or 1 seconds

(ii) Find the acceleration of the dot when it first comes to instantaneous rest. [2]

$a = \frac{dv}{dt} = 6t - 12$

at $t = 1 \quad a = 6 - 12 = -6 \text{ m/s}^2$

(iii) Explain clearly why the total distance travelled by the dot in the interval $t = 0$ to $t = 4$ is **not** obtained by finding the value of s when $t = 4$. [2]

The value of s when $t=4$ gives the displacement of the dot from fixed point O. The total distance is equal to this value of s if the dot did not change its direction of travel from $t=0$ to $t=4$. However, when $t=2$, $v=-3$m/s and $t=4$, $v=9$m/s clearly indicates the dot changes its direction of travel. Hence the value of s when $t=2$ is not the total distance travelled by the dot.

(iv) Find the total distance travelled by the dot in the interval $t = 0$ to $t = 4$.	[3]

Total dist : $\int_0^1 v\,dt + \int_1^3 |v|\,dt + \int_3^4 v\,dt$

$= \left[t^3 - 6t^2 + 9t\right]_0^1 + \left|\left[t^3 - 6t^2 + 9t\right]_1^3\right| + \left[t^3 - 6t^2 + 9t\right]_3^4$

$= 4 + |(0 - 4)| + 4$

$= 12$ cm

12 It is given that $f(x) = 2\sin 2x$ and $g(x) = 3\cos\left(\frac{x}{2}\right) - 1$.	
(i) State the least and greatest values of $f(x)$.	[1]

least value of $f(x)$: -2

greatest value of $f(x)$: 2

(ii) State the least and greatest values of $g(x)$.	[2]

least value of $g(x)$: -4

greatest value of $g(x)$: 2

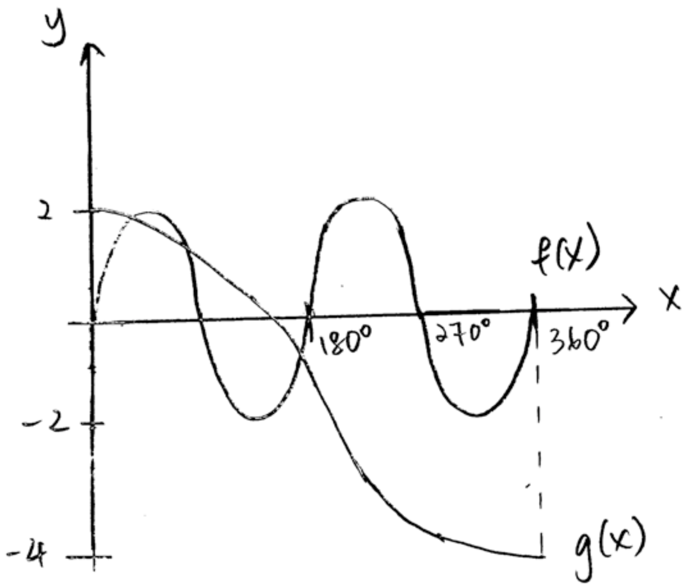
(iii) State the period of $f(x)$.	[1]

period of $f(x)$: π

(iv) State the period of $g(x)$.	[1]

period of $g(x)$: 4π

(v) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0^\circ \leqslant x \leqslant 360^\circ$. [4]



(vi) State the number of solutions of the equation $2\sin 2x + 1 = 3\cos\left(\frac{x}{2}\right)$ for $0^\circ \leqslant x \leqslant 360^\circ$. [1]

No of solutions : 3 #