2018 GCE O'Level

Additional Mathematics Paper 2 (4047/02)

Suggested Answers

1. (i) $x^2+3x+5=0$

Sum: $\alpha+\beta=-3$

Product: $\alpha\beta=5$

$(\alpha+1)(\beta+1)=\alpha\beta+\alpha+\beta+1$

$=5-3+1=3$ (shown)

(ii) $\text{Sum}_{new}:\dfrac{2}{\alpha+1}+\dfrac{2}{\beta+1}=\dfrac{2(\beta+\alpha)+4}{3}=-\dfrac{2}{3}$

$\text{Product}_{new}:\dfrac{2}{\alpha+1}\times\dfrac{2}{\beta+1}=\dfrac{4}{3}$

Equation: $3x^2+2x+4=0$

2. $(1-4x)(2+ax)^6=(1-4x)\left(2^6+6(2)^5ax+15(2)^4(ax)^2+\ldots\right)$

$=64+(192a-256)x+(240a^2-768a)x^2$

$\therefore -160=192a-256$

$a=\dfrac{1}{2}$

$240a^2-768a=b$

$b=444$

3. $\angle CQP=180°-\angle AQP$

$\angle QPC=180°-\angle CQP-\angle BCP$

$=180°-(180°-\angle AQP)-\angle BCP$

$=\angle AQP-\angle BCP$

Now, $\angle BPQ=\angle APQ-\angle APB$

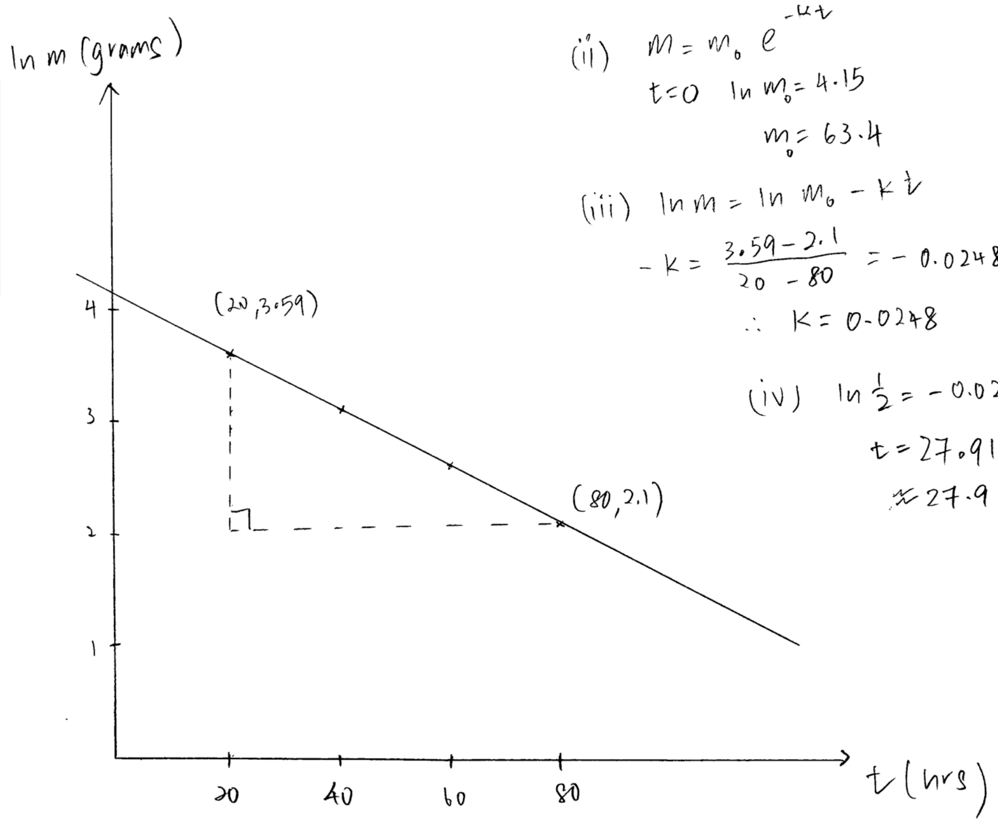
$=\angle AQP-\angle APB \quad \because AP=AQ$

$=\angle AQP-\angle BCP \quad \because \angle APB=\angle BCP$ (Alternate Segment Theorem)

$=\angle QPC$

$\therefore PQ$ bisect $\angle BPC \because \angle BPQ=\angle QPC$

4.



(ii) $m = m_0 e^{-kt}$

$t = 0 \quad \ln m_0 = 4.15$

$m_0 = 63.4$

(iii) $\ln m = \ln m_0 - kt$

$-k = \dfrac{3.59 - 2.1}{20 - 80} = -0.02483$

$\therefore k = 0.0248$

(iv) $\ln \frac{1}{2} = -0.02483t$

$t = 27.91$

≈ 27.9 hrs

5. (i)
$BC = 800\tan\theta$
$DC = 1200 - BC$
$\therefore CD = 1200 - 800\tan\theta$

(ii)
$$\cos\theta = \frac{ED}{CD}$$
$$DE = (1200 - 800\tan\theta)\cos\theta$$
$$= 1200\cos\theta - 800\sin\theta$$

(iii)
$$1200\cos\theta - 800\sin\theta = R\cos(\theta+\alpha)$$
$$= R\cos\alpha\cos\theta - R\sin\alpha\sin\theta$$
$1200 = R\cos\alpha$
$800 = R\sin a$
$$\tan\alpha = \frac{2}{3}$$
$\alpha = 33.7°$
$$\therefore DE = \sqrt{1200^2 + 800^2}\cos(\theta + 33.7°) = 200$$
$\theta = 48.3°$

6. (i)
$$\frac{d}{dx}x\cos x = \cos x - x\sin x$$

(ii)
$$\int \cos x - x\sin x\,dx = x\cos x + c$$
$$\int x\sin x\,dx = \int \cos x\,dx - x\cos x + c$$
$$= \sin x - x\cos x + c$$

(iii)
$$\frac{d}{dx}x^2\sin x = 2x\sin x - x^2\cos x$$
$$\int 2x\sin x - x^2\cos x\,dx = x^2\sin x + c$$
$$\int x^2\cos x\,dx = \int 2x\sin x\,dx - x^2\sin x + c$$
$$= 2(\sin x - x\cos x) - x^2\sin x + c$$
$$= 2\sin x - 2x\cos x - x^2\sin x + c$$

7. (i)

$$d = 840\left(1 - e^{-\frac{t}{80}}\right) - 2t$$

$$= 840 - 840e^{-\frac{t}{80}} - 2t$$

$$v = \frac{21}{2}e^{-\frac{t}{80}} - 2$$

$$t = 10, \quad s = 7.266 \approx 7.27 \text{ m/s}$$

$$a = -\frac{21}{160}e^{-\frac{t}{80}}$$

$$t = 10, \quad a = -0.1158 \approx -0.116 \text{ m/s}^2$$

(ii) Negative sign means deceleration, i.e. her speed is decreasing.

(iii)

$$1.5 = \frac{21}{2}e^{-\frac{t}{80}} - 2$$

$$e^{-\frac{t}{80}} = \frac{1}{3}$$

$$t = 87.888$$

$$d = 840\left(1 - \frac{1}{3}\right) - 2(87.888) = 384.22$$

$$500 - 384.22 = 115.77 \approx 116 \text{ m}$$

8. (i) $p(x) = 2x^3 + 5x^2 - 18$

By Remainder Theorem, $p(-2) = -14$

(ii) By Factor Theorem, $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 + 5\left(\frac{3}{2}\right)^2 - 18 = 0$

$\therefore (2x-3)$ is a factor.

(iii) $2x^3 + 5x^2 - 18 = (2x-3)(x^2 + ax + 6)$

Comparing coefficient of x^2 :

$5 = 2a - 3$
$a = 4$

Consider $x^2 + 4x + 6 = 0$

$D = b^2 - 4ac = 16 - 4(1)(6) = -8 < 0$
$\therefore 2x^3 + 5x^2 - 18 = 0$ has only 1 real root.

(iv) $2^{3y+1} + 5(2^{2y}) = 18$

$2(2^y)^3 + 5(2^y)^2 - 18 = 0$

Let $2^y = x$.

$\Rightarrow 2x^3 + 5x^2 - 18 = 0$

Then from (ii) and (iii), since $x = \frac{3}{2}$ is the only real solution for $2x^3 + 5x^2 - 18 = 0$,

then $2^y = \frac{3}{2}$

$y = \frac{\ln\frac{3}{2}}{\ln 2} = 0.5849 \approx 0.585$

9. (i) $y = 2x^2 + (k+2)x + k$

$k = 5, \quad y = 2x^2 + 7x + 5$

$y = 19x - 13$

$2x^2 + 7x + 5 = 19x - 13$
$2x^2 - 12x + 18 = 0$
$D = b^2 - 4ac = 0$
$\because D = 0$, then $y = 19x - 13$ is tangent to $y = 2x^2 + 7x + 5$
$x = 3,\ y = 44$

(ii) Since coefficient of x^2 is 2,

$y = 2x^2 + (k+2)x + k$ is a positive curve.

For $y \le 0$, there must be at least 1 real root.

$b^2 - 4ac \ge 0$
$(k+2)^2 - 4(2)(k) \ge 0$
$(k-2)^2 \ge 0$

Hence for the case when k is exactly 2, $b^2 - 4ac = 0$ and $y \ge 0$.

Therefore there is only 1 value for k which y cannot be negative.

10. (i) $\frac{dy}{dx} = -3(7-3x)^{-\frac{1}{2}}$

Grad of Normal: $\frac{(7-3k)^{\frac{1}{2}}}{3}$

Equation of Normal at P:

$y - 2\sqrt{7-3k} = \frac{(7-3k)^{\frac{1}{2}}}{3}(x-k)$

Sub in $(-5,\ 0)$

$-2\sqrt{7-3k} = \frac{(7-3k)^{\frac{1}{2}}}{3}(-5-k)$

$6 = -(-5-k)$

$k = 1$

(ii) Equation of tangent

$y - 4 = -\frac{3}{2}(x-1)$

$y = -\frac{3}{2}x + \frac{11}{2}$

when $y = 0$, $x = \frac{11}{3}$

(iii) Area:

$= \frac{1}{2}\left(\frac{11}{3} - 1\right)4 - \int_1^{7/3} 2\sqrt{7-3x}\ dx$

$= \frac{16}{3} - \left[-\frac{4}{9}(7-3x)^{3/2}\right]_1^{7/3} = \frac{16}{9} units^2$

11. (i) $Grad_{AB} = \frac{8-4}{9-1} = \frac{1}{2}$ $\quad Grad_{BC} = \frac{12-8}{7-9} = -2$

$Grad_{AB} \times Grad_{BC} = -1$

Hence, $\angle ABC = 90°$

(ii) Since angle in semi circle is equals to 90 degrees, and $\angle ABC = 90°$, it follows that triangle ABC lies in a circle with AC as the diameter.

(iii) Since AC is the diameter, then the centre of circle is the mid point of AC. Denoting M as mid point of AC, we have

$$M\left(\frac{7+1}{2}, \frac{4+12}{2}\right) = M(4, 8)$$

Noting further that the y – coordinate of M and B are the same, then its clear that radius of the circle is 5 units.

Equation: $(x-4)^2 + (y-8)^2 = 5^2 = 25$

(iv) Since the y – coordinate of M and B are the same, then MB is a horizontal line parallel to the x – axis. Hence, tangent at point B will be perpendicular to MB, which is parallel to the y – axis.

(v) $Grad_{MC} = \frac{12-8}{7-4} = \frac{4}{3}$

Equation of tangent at C:

$$y - 12 = -\frac{3}{4}(x-7)$$

$$y = -\frac{3}{4}x + \frac{69}{4}$$