2018 GCE O'Level

Additional Mathematics Paper 1 (4047/01)

Suggested Answers

1. $\sqrt{125^x} = \frac{5 \cdot 5^{-x}}{25}$

$125^{\frac{x}{2}} \cdot 5^x = \frac{1}{5}$

$3125^{\frac{x}{2}} = \frac{1}{5}$

$\frac{x}{2} \ln 3125 = \ln \frac{1}{5}$

$\frac{x}{2} = \frac{\ln \frac{1}{5}}{\ln 3125} = -\frac{1}{5}$

$125^{\frac{x}{2}} = 0.3807 \approx 0.381$

2. (i) $\tan C = \tan\left(180° - \left(A + B\right)\right)$

$= \frac{\tan 180° - \tan\left(A + B\right)}{1 + \left(\tan 180°\right)\left(\tan\left(A + B\right)\right)}$

$= -\tan\left(A + B\right)$

(ii) $\tan C = -\left[\frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60°}\right]$

$= -\left[\frac{1 + \sqrt{3}}{1 - \sqrt{3}}\right]$

$\frac{1 + \sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{\sqrt{3} + 1 + 3 + \sqrt{3}}{3 - 1}$

$= \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$

3. $\frac{7x^2 - 12x + 17}{\left(2x - 1\right)\left(x^2 + 4\right)} = \frac{A}{2x - 1} + \frac{Bx + C}{x^2 + 4}$

$7x^2 - 12x + 17 = A\left(x^2 + 4\right) + \left(Bx + C\right)\left(2x - 1\right)$

Let $x = \frac{1}{2}$

$\frac{7}{4} - 6 + 17 = A\left(\frac{1}{4} + 4\right)$

$A = 3$

Compare coefficient of x^2

$7 = A + 2B$

$B = 2$

Compare coefficient of x^0

$17 = 4A - C$

$C = -5$

$\frac{7x^2 - 12x + 17}{\left(2x - 1\right)\left(x^2 + 4\right)} = \frac{3}{2x - 1} + \frac{2x - 5}{x^2 + 4}$

4. (a) $3 + 2\sqrt{5} = \frac{6 + \sqrt{80}}{2}$

$\therefore \frac{-a + \sqrt{a^2 - 4b}}{2} \equiv \frac{6 + \sqrt{80}}{2}$

$a = -6$

$a^2 - 4b = 80$

$b = -11$

(b) Breadth $= \frac{24 + \sqrt{48}}{6 + \sqrt{12}}$

$= \frac{12 + 2\sqrt{3}}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$

$= \frac{30 - 6\sqrt{3}}{6} = 5 - \sqrt{3}$

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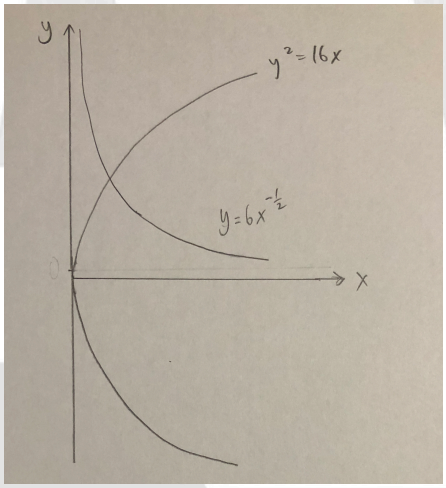
5. (i)



(ii) $y^2 = 16x$ $\qquad y = \dfrac{6}{\sqrt{x}}$

$$\Rightarrow \left(\frac{6}{\sqrt{x}}\right)^2 = 16x$$

$$x^2 = \frac{36}{16}$$

$$x = \frac{3}{2} \quad \text{or} \quad -\frac{3}{2}\left(\text{rej} \because x > 0\right)$$

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6. (i) $\log_3 x + \dfrac{\log_3 x}{\log_3 9} = \log_3 x + \dfrac{1}{2}\log_3 x$

$= \dfrac{3}{2}\log_3 x$

$= \dfrac{3\lg x}{2\lg 3}$

(ii) $\dfrac{3\lg x}{2\lg 3} = 4$

$3\lg x = 8\lg 3$

$\lg x = \dfrac{8}{3}\lg 3$

$x = 3^{\frac{8}{3}} = 18.72 \approx 18.7$

7. (i) Volume at $x = 9$,

$V = \left[\dfrac{1}{3}\pi\left(9^2\right)\right]\left[36 - 9\right] = 729\pi$

Time taken $= \dfrac{729\pi}{18\pi} = 40.5$ seconds

(ii) $\dfrac{dV}{dx} = \left(\dfrac{2}{3}\pi x\right)\left(36 - x\right) + \left(-1\right)\left(\dfrac{1}{3}\pi x^2\right)$

When $x = 9$, $\dfrac{dV}{dx} = 135\pi$

$\dfrac{dV}{dt} = \dfrac{dV}{dx} \times \dfrac{dx}{dt}$

$18\pi = 135\pi\left(\dfrac{dx}{dt}\right)$

$\dfrac{dx}{dt} = \dfrac{2}{15}$ cm/s

8. (i) $8\sin^2 x + 2\cos^2 x = 6\sin^2 x + 2\sin^2 x + 2\cos^2 x$

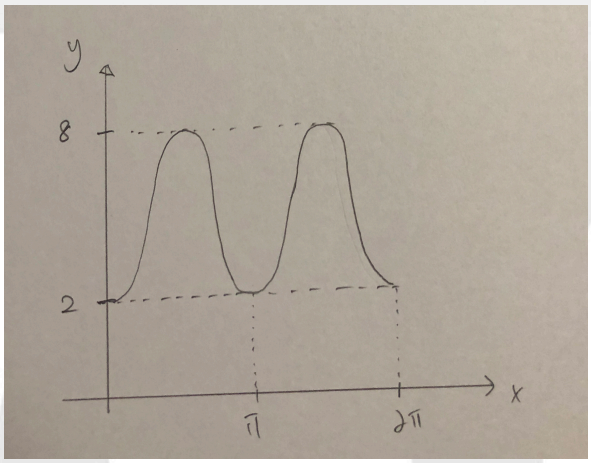
$= 3 - 3\cos 2A + 2$

$= 5 - 3\cos 2A$

(ii) Amplitude: 3

Period: π

(iii)



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9. (i) Gradient of AC $= \dfrac{10-0}{0-5} = -2$

Gradient of OB $= \dfrac{1}{2}$

$\because$ OB pass through origin

Equation of OB: $y = \dfrac{1}{2}x$

(ii) Let M be the midpoint of OB.

Equation of AC: $y = -2x + 10$

$-2x + 10 = \dfrac{1}{2}x$

$x = 4, \quad y = 2$

$\therefore M(4,2)$

Let point B be (x, y)

$(4,2) = \left(\dfrac{0+x}{2}, \dfrac{0+y}{2}\right)$

$\therefore x = 8, \quad y = 4$

$B(8,4)$

10. (i) Circumference: $20 - 3x = 2\pi r$ therefore $r = \dfrac{20-3x}{2\pi}$

(ii) Total Area: $\dfrac{1}{2}x^2 \sin 60° + \pi\left[\dfrac{(20-3x)}{2\pi}\right]^2 = \dfrac{\sqrt{3}\pi x^2 + (20-3x)^2}{4\pi}$

$\dfrac{dA}{dx} = \dfrac{1}{4\pi}\left(2\sqrt{3}\pi x + 2(20-3x)(-3)\right)$

when $\dfrac{dA}{dx} = 0 \qquad x = 4.15$

(iii) $\dfrac{d^2A}{dx^2} = \dfrac{1}{4\pi}\left(2\sqrt{3}\pi + 18\right) > 0$

Hence, area is a minimum.

Therefore, gardener will be disappointed.

11. (i) By long division

$$f'(x) = 5 + \frac{6}{2x-3}$$

(ii) $x > \frac{3}{2}$

$2x > 3$

$2x - 3 > 0$

then $\frac{6}{2x-3} > 0$

hence $f'(x) = 5 + \frac{6}{2x-3} > 0$ for $x > \frac{3}{2}$

Therefore $f(x)$ is an increasing function

(iii) $f''(x) = -\frac{6}{(2x-3)^2}$

Since $(2x-3)^2 > 0$ for all x

Then $-\frac{6}{(2x-3)^2} < 0$

Hence, $f'(x)$ is a decreasing function.

(iv) $f(x) = 5x + 3\ln(2x-3) + c$, $x > \frac{3}{2}$

$f(2) = 8$ then $c = -2$

$f(x) = 5x + 3\ln(2x-3) - 2$, $x > \frac{3}{2}$

12. (i) Since PQ : AR is 1 : 4

By similar triangle:

R (8, 0)

As graph is symmetrical about $x = 8$

Then x-coordinate of S will be 8+10=18

Since S lies on y = 15

Then S $(18,\ 15)$

(ii) Let coordinate Q be (0,b)

Then by similar triangle

$$\frac{15-b}{b} = \frac{1}{4}$$

$$b = 12$$

Gradient of SR $= a = \frac{15-0}{18-8} = \frac{3}{2}$