## 2018 GCE O'Level

## Additional Mathematics Paper 2 (4047/02)

## Suggested Answers

1. (i) $x^{2}+3 x+5=0$

Sum: $\alpha+\beta=-3$
Product: $\alpha \beta=5$

$$
\begin{aligned}
(\alpha+1)(\beta+1) & =\alpha \beta+\alpha+\beta+1 \\
& =5-3+1=3 \text { (shown) }
\end{aligned}
$$

(ii)
$\operatorname{Sum}_{\text {new }}: \frac{2}{\alpha+1}+\frac{2}{\beta+1}=\frac{2(\beta+\alpha)+4}{3}=-\frac{2}{3}$
Product $_{\text {new }}: \frac{2}{\alpha+1} \times \frac{2}{\beta+1}=\frac{4}{3}$
Equation: $3 x^{2}+2 x+4=0$
2. $(1-4 x)(2+a x)^{6}=(1-4 x)\left(2^{6}+6(2)^{5} a x+15(2)^{4}(a x)^{2}+\ldots\right)$

$$
=64+(192 a-256) x+\left(240 a^{2}-768 a\right) x^{2}
$$

$\therefore-160=192 a-256 \quad 240 a^{2}-768 a=b$
$a=\frac{1}{2}$
3. $\angle C Q P=180^{\circ}-\angle A Q P$
$\angle Q P C=180^{\circ}-\angle C Q P-\angle B C P$

$$
\begin{aligned}
& =180^{\circ}-\left(180^{\circ}-\angle A Q P\right)-\angle B C P \\
& =\angle A Q P-\angle B C P
\end{aligned}
$$

Now, $\angle B P Q=\angle A P Q-\angle A P B$

$$
\begin{aligned}
& =\angle A Q P-\angle A P B \quad \because A P=A Q \\
& =\angle A Q P-\angle B C P \\
& =\angle Q P C
\end{aligned}
$$

$\therefore P Q$ bisect $\angle B P C \because \angle B P Q=\angle Q P C$

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## Hear what our students have to say...

## Eve Lee Shi, NJC

${ }^{66}$ Chu Wei is a whole package deal of teacher, mentor and personal cheerleader hitting you all at once. he's so much more than just your tuition teacher with a mission to
 reteach what you should have learnt in school. He is one of the rare few tutors I have ever had to be as concerned as he is about his students' welfare, and their prospective careers.

## Ooi Qiu Min, NYJC

${ }^{6}$ Chu Wei's tuition provides a very personalised experience whereby, he will point out each of our mistakes and ensure that we will not repeat it in our exams or future practices.
 Unlike the other tutors, he is like an encouraging friend who believes in our potential and uses his patience to guide us instead of stressing us.

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5. (i) $B C=800 \tan \theta$
$D C=1200-B C$
$\therefore C D=1200-800 \tan \theta$
(ii)

$$
\begin{aligned}
\cos \theta & =\frac{E D}{C D} \\
D E & =(1200-800 \tan \theta) \cos \theta \\
& =1200 \cos \theta-800 \sin \theta
\end{aligned}
$$

(iii) $1200 \cos \theta-800 \sin \theta=R \cos (\theta+\alpha)$
$=R \cos \alpha \cos \theta-R \sin \alpha \sin \theta$
$1200=R \cos \alpha$
$800=R \sin a$
$\tan \alpha=\frac{2}{3}$
$\alpha=33.7^{\circ}$
$\therefore D E=\sqrt{1200^{2}+800^{2}} \cos \left(\theta+33.7^{\circ}\right)=200$
$\theta=48.3^{\circ}$
6. (i) $\frac{d}{d x} x \cos x=\cos x-x \sin x$
(ii) $\int \cos x-x \sin x d x=x \cos x+c$
$\int x \sin x d x=\int \cos x d x-x \cos x+c$ $=\sin x-x \cos x+c$
(iii) $\frac{d}{d x} x^{2} \sin x=2 x \sin x-x^{2} \cos x$
$\int 2 x \sin x-x^{2} \cos x d x=x^{2} \sin x+c$
$\int x^{2} \cos x d x=\int 2 x \sin x d x-x^{2} \sin x+c$

$$
\begin{aligned}
& =2(\sin x-x \cos x)-x^{2} \sin x+c \\
& =2 \sin x-2 x \cos x-x^{2} \sin x+c
\end{aligned}
$$

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## Suggested Answers

7. (i)

$$
\begin{aligned}
& \begin{aligned}
d & =840\left(1-e^{-\frac{t}{80}}\right)-2 t \\
& =840-840 e^{-\frac{t}{80}}-2 t \\
v & =\frac{21}{2} e^{-\frac{t}{80}}-2 \\
t & =10, \quad s=7.266 \approx 7.27 \mathrm{~m} / \mathrm{s} \\
a & =-\frac{21}{160} e^{-\frac{t}{80}} \\
t & =10, \quad a=-0.1158 \approx-0.116 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
\end{aligned}
$$

(ii) Negative sign means deceleration, i.e. her speed is decreasing.
(iii)

$$
\begin{aligned}
& 1.5=\frac{21}{2} e^{-\frac{t}{80}}-2 \\
& e^{-\frac{t}{80}}=\frac{1}{3} \\
& t=87.888 \\
& d=840\left(1-\frac{1}{3}\right)-2(87.888)=384.22 \\
& 500-384.22=115.77 \approx 116 \mathrm{~m}
\end{aligned}
$$

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## Suggested Answers

8. (i) $p(x)=2 x^{3}+5 x^{2}-18$

By Remainder Theorem, $\quad p(-2)=-14$
(ii) By Factor Theorem, $\quad p\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^{3}+5\left(\frac{3}{2}\right)^{2}-18=0$
$\therefore(2 x-3)$ is a factor.
(iii) $2 x^{3}+5 x^{2}-18=(2 x-3)\left(x^{2}+a x+6\right)$

Comparing coefficient of $x^{2}$ :
$5=2 a-3$
$a=4$
Consider $x^{2}+4 x+6=0$
$D=b^{2}-4 a c=16-4(1)(6)=-8<0$
$\therefore 2 x^{3}+5 x^{2}-18=0$ has only 1 real root.
(iv) $2^{3 y+1}+5\left(2^{2 y}\right)=18$
$2\left(2^{y}\right)^{3}+5\left(2^{y}\right)^{2}-18=0$
Let $2^{y}=x$.
$\Rightarrow 2 x^{3}+5 x^{2}-18=0$
Then from (ii) and (iii), since $x=\frac{3}{2}$ is the only real solution for $2 x^{3}+5 x^{2}-18=0$,
then $2^{y}=\frac{3}{2}$
$y=\frac{\ln \frac{3}{2}}{\ln 2}=0.5849 \approx 0.585$

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## Suggested Answers

9. (i) $y=2 x^{2}+(k+2) x+k$
$k=5, \quad y=2 x^{2}+7 x+5$
$y=19 x-13$
$2 x^{2}+7 x+5=19 x-13$
$2 x^{2}-12 x+18=0$
$D=b^{2}-4 a c=0$
$\because D=0$, then $y=19 x-13$ is tangent to $y=2 x^{2}+7 x+5$
$x=3, y=44$
(ii) Since coefficient of $x^{2}$ is 2,
$y=2 x^{2}+(k+2) x+k$ is a positive curve.

For $y \leq 0$, there must be at least 1 real root.
$b^{2}-4 a c \geq 0$
$(k+2)^{2}-4(2)(k) \geq 0$
$(k-2)^{2} \geq 0$
Hence for the case when $k$ is exactly $2, b^{2}-4 a c=0$ and $y \geq 0$.
Therefore there is only 1 value for $k$ which $y$ cannot be negative.

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## Suggested Answers

10. (i) $\frac{d y}{d x}=-3(7-3 x)^{-\frac{1}{2}}$

Grad of Normal: $\frac{(7-3 k)^{\frac{1}{2}}}{3}$
Equation of Normal at P :
$y-2 \sqrt{7-3 k}=\frac{(7-3 k)^{\frac{1}{2}}}{3}(x-k)$
Sub in $(-5,0)$
$-2 \sqrt{7-3 k}=\frac{(7-3 k)^{\frac{1}{2}}}{3}(-5-k)$
$6=-(-5-k)$
$k=1$
(ii) Equation of tangent
$y-4=-\frac{3}{2}(x-1)$
$y=-\frac{3}{2} x+\frac{11}{2}$
when $y=0, x=\frac{11}{3}$
(iii) Area:
$=\frac{1}{2}\left(\frac{11}{3}-1\right) 4-\int_{1}^{7 / 3} 2 \sqrt{7-3 x} d x$
$=\frac{16}{3}-\left[-\frac{4}{9}(7-3 x)^{3 / 2}\right]_{1}^{7 / 3}=\frac{16}{9}$ units $^{2}$

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## Suggested Answers

11. (i)
$\operatorname{Grad}_{A B}=\frac{8-4}{9-1}=\frac{1}{2} \quad \operatorname{Grad}_{B C}=\frac{12-8}{7-9}=-2$
$\operatorname{Grad}_{A B} \times \operatorname{Grad}_{B C}=-1$
Hence, $\angle A B C=90^{\circ}$
(ii) Since angle in semi circle is equals to 90 degrees, and $\angle A B C=90^{\circ}$, it follows that triangle ABC lies in a circle with AC as the diameter.
(iii) Since AC is the diameter, then the centre of circle is the mid point of AC. Denoting M as mid point of AC, we have
$M\left(\frac{7+1}{2}, \frac{4+12}{2}\right)=M(4,8)$
Noting further that the $y$-coordinate of $M$ and $B$ are the same, then its clear that radius of the circle is 5 units.

Equation: $(x-4)^{2}+(y-8)^{2}=5^{2}=25$
(iv) Since the $y$-coordinate of $M$ and $B$ are the same, then $M B$ is a horizontal line parallel to the $x$-axis. Hence, tangent at point B will be perpendicular to MB , which is parallel to the $y$-axis.
(v)
$\operatorname{Grad}_{M C}=\frac{12-8}{7-4}=\frac{4}{3}$
Equation of tangent at C :
$y-12=-\frac{3}{4}(x-7)$
$y=-\frac{3}{4} x+\frac{69}{4}$

