2017 GCE O'Level
Additional Mathematics Paper 1 (4047/01)

Suggested Answers

1. $\frac{d^2y}{dx^2} = 8 - 6x$

$\frac{dy}{dx} = 8x - 3x^2 + c$

$\left.\frac{dy}{dx}\right|_{x=2} = 3$

$\therefore 8(2) - 3(2)^2 + c = 3$

$16 - 12 + c = 3$

$c = -1$

$\Rightarrow \frac{dy}{dx} = 8x - 3x^2 - 1$

$y = 4x^2 - x^3 - x + d$

Sub in $(2,8)$

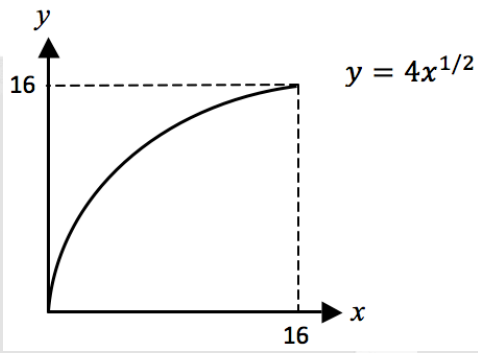
$8 = 4(2)^2 - (2)^3 - (2) + d$

$8 = 16 - 8 - 2 + d$

$d = 2$

$\therefore$ Equation of curve $y = -x^3 + 4x^2 - x + 2$

2. (i)



(ii) $4y = 7x + 4$ ----- (1)

$y = 4x^{\frac{1}{2}}$ ----- (2)

Sub (2) in (1)

$4\left(4x^{\frac{1}{2}}\right) = 7x + 4$

$16x^{\frac{1}{2}} = 7x + 4$

Suggested Answers

$7x - 16x^{\frac{1}{2}} + 4 = 0$

Let $x^{\frac{1}{2}} = w$

$7w^2 - 16w + 4 = 0$

$(w-2)(7w-2) = 0$

$w = 2 \quad \text{or} \quad w = \frac{2}{7}$

$\therefore x^{\frac{1}{2}} = 2 \qquad x^{\frac{1}{2}} = \frac{2}{7}$

$x = 4 \qquad x = \frac{4}{49}$

$y = 8 \qquad y = \frac{8}{7}$

$\therefore$ Coordinates are $(4,8)$ and $\left(\frac{4}{49}, \frac{8}{7}\right)$.

3. $\frac{1}{y} = \frac{1}{\sqrt{x}} m + c$

$\frac{1}{0.25} = \frac{1}{\sqrt{0.04}} m + c$

$4 - 5m = c \quad$ ---- (1)

$\frac{1}{0.5} = \frac{1}{\sqrt{1.00}} m + c$

$2 - m = c \quad$ ---- (2)

(1) = (2)

$4 - 5m = 2 - m$

$m = \frac{1}{2}$

$c = \frac{3}{2}$

$\Rightarrow \frac{1}{y} = \frac{1}{\sqrt{x}}\left(\frac{1}{2}\right) + \frac{3}{2}$

$\frac{1}{y} = \frac{5}{3}$

$\therefore y = \frac{3}{5}$

Suggested Answers

4. $7x^2 - 3x + 1 = 0$

Sum of roots: $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{7}$

$$\frac{\alpha + \beta}{\alpha\beta} = \frac{3}{7} \quad \text{---- (1)}$$

Product of roots: $\frac{1}{\alpha\beta} = \frac{1}{7}$

$$\alpha\beta = 7 \quad \text{---- (2)}$$

From (1): $\alpha + \beta = 3$

Sum of new roots: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = -5$

Product of new roots: $\alpha^2\beta^2 = (\alpha\beta)^2 = 7^2 = 49$

$\therefore$ Equation: $x^2 + 5x + 49 = 0$

5. (i) LHS: $\dfrac{\sec x + \operatorname{cosec} x}{\sec x - \operatorname{cosec} x}$

$$= \frac{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}{\dfrac{1}{\cos x} - \dfrac{1}{\sin x}}$$

$$= \frac{\sin x + \cos x}{\sin x \cos x} \div \frac{\sin x - \cos x}{\sin x \cos x}$$

$$= \frac{\sin x + \cos x}{\sin x - \cos x}$$

$$= \frac{\cos x\left(\dfrac{\sin x}{\cos x} + 1\right)}{\cos x\left(\dfrac{\sin x}{\cos x} - 1\right)} = \frac{\tan x + 1}{\tan x - 1} = \text{RHS} \quad \text{(shown)}$$

(ii) $\dfrac{\tan x + 1}{\tan x - 1} = \dfrac{5}{2}$

$2\tan x + 2 = 5\tan x - 5$

$\tan x = \dfrac{7}{3}$

$\alpha = 1.1659$

$x = 1.17 \text{ rad}, \quad 4.31 \text{ rad}$

6. (i) Total length: $4y + 6xx = 288$

$4y = 288 - 6x$

$y = 72 - \dfrac{3}{2}x$

Total area: $A = 3xy$

$$= 3x\left(72 - \frac{3}{2}x\right) = 216x - \frac{9}{2}x^2 \quad \text{(shown)}$$

(ii) $\dfrac{dA}{dx} = 216 - 9x = 0$

$x = 24$

$y = 36$

$\therefore$ Dimensions of each tennis court is 36 m by 24 m.

Suggested Answers

7. (i) Area of $\Delta ABC : \frac{1}{2}(AB)(AC)\sin 60° = \frac{1}{4}\left(9+\sqrt{3}\right)$

$$\frac{1}{2}\left(\sqrt{3}+1\right)(AC)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4}\left(9+\sqrt{3}\right)$$

$$AC = \frac{9+\sqrt{3}}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{24-6\sqrt{3}}{6} = 4-\sqrt{3}$$

$$\therefore a = 4, \quad b = -1$$

(ii) $BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 60°$

$$= \left(1+\sqrt{3}\right)^2 + \left(4-\sqrt{3}\right)^2 - \left(1+\sqrt{3}\right)\left(4-\sqrt{3}\right)$$

$$= 1+2\sqrt{3}+3+16-8\sqrt{3}+3-4-3\sqrt{3}+3$$

$$= 22-9\sqrt{3} \text{ cm}^2$$

8. (i) By long division,

$$3x^3 - x^2 + 27x - 9 = \left(x^2+9\right)(3x-1)$$

(ii) $$\frac{6+11x-5x^2}{\left(x^2+9\right)(3x-1)} \equiv \frac{Ax+B}{x^2+9} + \frac{C}{3x-1}$$

$$\Rightarrow 6+11x-5x^2 = (Ax+B)(3x-1) + C\left(x^2+9\right)$$

Sub $x = \frac{1}{3}$

$$6+11\left(\frac{1}{3}\right) - 5\left(\frac{1}{3}\right)^2 = C\left(\left(\frac{1}{3}\right)^2 + 9\right)$$

$$C = 1$$

Compare coefficient of x^2: $-5 = 3A + C$

$$A = -2$$

Compare coefficient of x^0: $6 = -B + 9C$

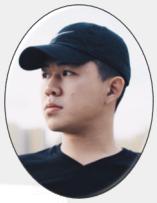
$$B = 3$$

$$\therefore \frac{6+11x-5x^2}{\left(x^2+9\right)(3x-1)} = \frac{3-2x}{x^2+9} + \frac{1}{3x-1}$$

Suggested Answers

Hear what our students have to say…

Yeo Dong Han,
Saint Andrews Junior College



During my time in J1, I had difficulties understanding Math problems and as a result, my grades fell much below what I expected. I could still remember failing math examinations then. It was during the mid of J1 when my friend introduced me to this tuition class. During a short period of 2-3 months, my results took a great turn. Eventually, I managed to achieve an A in my promos and my A Levels Math Exam. I credit this achievement mostly to my math tutor, Chu Wei. He is dedicated in guiding us and never stopped encouraging and motivating us by telling us that we are capable of scoring A. He takes great effort in making his math questions fun to do despite being challenging. This enabled me not only to take an interest in math, but also to be able to cope with difficult questions with ease. All in all, I am glad to have him as my tutor and as a friend. Thank You Chu Wei!

Quinn Chen Lu Shi,
Victoria Junior College

Whether you're the kind of 'a relatively doing well' but can never score an 'A' student or a 'weak in math' student like I was, please take some time to read the long but honest review:
Chuwei is a tutor with both great knowledge and a big heart. As a teacher, he never fails to clear our doubts and explain basic mathematical concepts which are crucial in cultivating problem solving skills. He goes the extra mile to answer our questions out of class and even late nights. As an Arts student who has always struggled with math, Chuwei has consistently guided me since J1 and allow me to gain confidence.
His depth of knowledge of the subject and the syllabus has enabled him to quickly identify his students' weaknesses and advise each of us individually on how we can improve. From failing math in the past, I've finally did well in A levels.
Honestly, it was never a smooth journey for me when it came to this daunting subject , and there were times where I faced setbacks in examinations. However, Chuwei, someone who believes in the best in every student, gave me the encouragement I needed each time and allowed me to push through until the end. He has been more than just a tutor to all of us.
To parents or students reading, if you want a tutor who is dedicated, whose teaching is effective or a tutor who can give your child/you maximum exposure (something all students really need in A level H2 mathematics unlike O levels), I believe Chuwei is the tutor you are looking for.
To students, no matter what tutor you're going to eventually, please remember that math really requires you to PRACTISE and don't be complacent. There are many 'A' students who eventually scored a 'B' just because they got lazy or are over confident. There are also those who used to fail but got an 'A' because they tried hard enough.

9. (i) $\frac{dv}{dt} = 10$

$v = 10t + c$

$t = 0,\ v = 0 \Rightarrow c = 0$

$\therefore t = 4,\ v = 10(4) = 40$ m/s

(ii) $v = 10t$

$s = 5t^2 + d$

Suggested Answers

When $t = 0,\ s = 0 \Rightarrow d = 0$

$\therefore t = 4,\ s = 5(4)^2 = 80$ m

(iii) $\dfrac{dV}{dT} = 10 - kT$

$V = 10T - \dfrac{k}{2}T^2 + c$

When $T = 0,\ V = 40$

$40 = 10(0) - \dfrac{k}{2}(0)^2 + c$

$c = 40$

When $T = 3,\ V = 0$

$0 = 10(3) - \dfrac{k}{2}(3)^2 + 40$

$k = \dfrac{140}{9}$ (shown)

10. (i) $\angle PBA = \angle ACB$ (Alternate segment theorem)

$= \angle DAC$ (AD parallel to BC) (shown)

(ii) $\angle CBT = \angle BAC$ (Alternate segment theorem)

$= \angle DAB - \angle DAC$

$= (180° - \angle DCB) - \angle DAC$ (Opp angles in cyclic quad)

$= 180° - \angle DCB - \angle PBA$ ($\because \angle DAC = \angle PBA$)

$= 180° - (\angle DCA + \angle ACB) - \angle PBA$

$= 180° - (90° + \angle ACB) - \angle PBA$ (Angles in semicircle)

$= 180° - (90° + \angle PBA) - \angle PBA$ (Alternate segment theorem)

$= 90° - 2(\angle PBA)$ (shown)

Suggested Answers

11. (i) $y = \dfrac{2x+1}{x-1}$

$$\frac{dy}{dx} = \frac{2(x-1)-(1)(2x+1)}{(x-1)^2}$$

$$= -\frac{3}{(x-1)^2} < 0 \quad \text{for all } x \in \mathbb{R}$$

Since $\dfrac{dy}{dx} \neq 0$ for all values of x, therefore the curve has no turning points.

(ii) $\dfrac{2x+1}{x-1} = 2 + \dfrac{3}{x-1}$

$$\int_2^4 2 + \frac{3}{x-1}\,dx = \left[2x + 3\ln|x-1|\right]_2^4$$

$$= (8 + 3\ln 3) - (4 - 3\ln 1)$$
$$= 4 + 3\ln 3$$

Area of trapezium $= \dfrac{1}{2}(5+3)(2) = 8$

Area of shaded region $= 8 - (4 + 3\ln 3)$

$= 0.704 \text{ units}^2$

12. (i) $x^2 + y^2 + 8x - 24y + 96 = 0$

$$(x+4)^2 - 16 + (y-12)^2 - 144 + 96 = 0$$

$$(x+4)^2 + (y-12)^2 = 8^2$$

Centre $(-4, 12)$ Radius=8

$3y + 4x = k$

$y = -\dfrac{4}{3}x + \dfrac{k}{3}$

Let general point on normal be $\left(x, -\dfrac{4}{3}x + \dfrac{k}{3}\right)$

Gradient: $\dfrac{12 - \left(-\frac{4}{3}x + \frac{k}{3}\right)}{-4 - x} = -\dfrac{4}{3}$

$$12+\frac{4}{3}x-\frac{k}{3}=\frac{16}{3}+\frac{4}{3}x$$

$$\therefore k=20$$

(ii) Sub $y=-\frac{4}{3}x+\frac{20}{3}$ in equation of circle

$$(x+4)^2+\left(-\frac{4}{3}x+\frac{20}{3}-12\right)^2=8^2$$

$$25x^2+200x-176=0$$

$$x=\frac{-200\pm\sqrt{200^2-4(25)(-176)}}{2(25)}$$

$$x=\frac{4}{5},\quad -\frac{44}{5}$$

$\because R$ is between S and centre of radius

$$\therefore x=\frac{4}{5},\quad y=-\frac{4}{3}\left(\frac{4}{5}\right)+\frac{20}{3}=\frac{28}{5}$$

$$S(5,0)$$

Distance $RS=\sqrt{\left(5-\frac{4}{5}\right)^2+\left(\frac{28}{5}\right)^2}=\sqrt{49}=7$ units