2017 GCE O'Level
Additional Mathematics Paper 2 (4047/02)

Suggested Answers

1. $y = e^{-x}x^2$

$$\frac{dy}{dx} = -e^{-x}x^2 + 2xe^{-x} = e^{-x}\left(-x^2 + 2x\right)$$

$$2\frac{dy}{dx} = e^{-x}\left(-2x^2 + 4x\right)$$

$$\frac{d^2y}{dx^2} = -e^{-x}\left(-x^2 + 2x\right) + e^{-x}\left(-2x + 2\right)$$

$$= e^{-x}\left(x^2 - 2x\right) + e^{-x}\left(2 - 2x\right)$$

$$= e^{-x}\left(x^2 - 4x + 2\right)$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}x^2 + e^{-x}\left(-2x^2 + 4x\right) + e^{-x}\left(x^2 - 4x + 2\right)$$

$$= 2e^{-x}$$

$$\therefore e^{x}\left(\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y\right) = 2$$

$$k = 2$$

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

2. (i) $\frac{d}{dx}\left(\tan x - x\right) = \sec^2 x - 1 = \tan^2 x$ (shown)

(ii) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x + 5\tan^2 x \, dx$

$$= \left[\tan x + 5\left(\tan x - x\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \left[\tan\frac{\pi}{3} + 5\left(\tan\frac{\pi}{3} - \frac{\pi}{3}\right)\right] - \left[\tan\frac{\pi}{6} + 5\left(\tan\frac{\pi}{6} - \frac{\pi}{6}\right)\right]$$

$$= \sqrt{3} + 5\sqrt{3} - \frac{5\pi}{3} - \left(\frac{\sqrt{3}}{3} + \frac{5\sqrt{3}}{3} - \frac{5\pi}{6}\right)$$

$$= 4\sqrt{3} - \frac{5}{6}\pi$$

$$\therefore a = 4, \quad b = -\frac{5}{6}$$

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

Blk 283, Bishan Street 22, #01-185, S(570283) Chu Wei – 9077 9077 www.aostudies.com.sg

3. (i) $T_{r+1} = {}^9C_r\left(px^3\right)^{9-r}\left(\frac{1}{x}\right)^r$

$= {}^9C_r\left(p^{9-r}x^{27-3r}\right)\left(x^{-r}\right)$

$= {}^9C_r \cdot p^{9-r}x^{27-4r}$

For $27-4r$ to be an even number,

$27-4r$ must be divisible by 2.

$\Rightarrow 27-4r = 2(13.5-2r)$

$\because 13.5-2r$ is not an integer, $27-4r$ cannot be divisible by 2, and thus cannot be an even number.

Hence, there are no even powers of x to this expansion.

(ii) $27-4r=11$ $\quad$ $27-4r=7$

$r=4$ $\quad$ $r=5$

$T_5 = T_{4+1} = {}^9C_4 \cdot p^{9-4}x^{27-4(4)} = 126p^5x^{11}$

$T_6 = T_{5+1} = {}^9C_5 \cdot p^{9-5}x^{27-4(5)} = 126p^4x^7$

$126p^5 = 2\left(126p^4\right)$

$p=2$

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

4. (i) $y = \frac{6}{\sqrt{x}} + x$

$\frac{dy}{dx} = 6\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)+1$

$= -\frac{3}{2x^{\frac{3}{2}}}+1=0$

$\frac{3}{2x^{\frac{3}{2}}}=1$

$\left(x^3\right)^{\frac{1}{2}}=3$

$x^3=9$ (shown)

Blk 283, Bishan Street 22, #01-185, S(570283) Chu Wei – 9077 9077 www.aostudies.com.sg

(ii) $\therefore x-$coordinate of $M = \sqrt[3]{9} \approx 2.08$

Gradient of A: $\left.\frac{dy}{dx}\right|_{x=1} = -\frac{3}{1}+1=-2$

Equation of tangent at A: $y-7=-2(x-1)$

$$y=-2x+9 \qquad \text{----- (1)}$$

Gradient of B: $\left.\frac{dy}{dx}\right|_{x=4} = -\frac{3}{4^{\frac{3}{2}}}+1=\frac{5}{8}$

Equation of tangent at B: $y-7=\frac{5}{8}(x-4)$

$$y=\frac{5}{8}x+\frac{9}{2} \qquad \text{----- (2)}$$

(1) = (2), $\quad -2x+9=\frac{5}{8}x+\frac{9}{2}$

$$x=\frac{12}{7}\approx 1.71$$

$\therefore x-$coordinate of P is less than $x-$coordinate of M.

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

Hear what our students have to say…

Eve Lee Shi,
Anderson Secondary School

I had only begun attending AO Studies classes in June during my fourth year in secondary school, and in the short span of a mere five months, my grades had skyrocketed; From borderline passes I struggled to produce, to the A1s I achieved in both A and E Maths at my Os.

It has truly been an exhilarating experience; not only does the tuition centre provide a perfectly conducive environment for learning, I found that I have also enjoyed interacting with fellow classmates and our teacher Chu Wei. He's wonderfully engaging and he never hesitates to share stories, opinions and jokes with the class.
Furthermore, the papers and the practices he provides us are challenging and creative. Through his lessons, I have not only learnt to tackle each and every math problem, I too have learnt to appreciate the fascinating complexity and flexibility of the beauty that is mathematics.

Leow Xingni,
Nanyang Junior College

I joined Chu Wei's tuition as a private candidate through a friend's recommendation, and to be honest I was pretty pessimistic about Math at first as it was my second attempt at the A levels. However, Chu Wei proved me wrong. His nightmarish and infuriating tutorials exposed me to a lot more different types of questions than normal school papers have and that helped to build my confidence when doing math. He stayed back after class to clear my doubts, replied my bombarding texts, answered my questions with patience and most importantly he made sure that I understood. He even went out of his way to help me with my university application, giving me guidance on the courses that I want to apply to.
He is a very driven teacher and a friend who will encourage and motivate you, even though sometimes he can come off a bit harsh and his honest advice may be unpleasant to hear, but rest assured he means well. And you'll get a slice of cake on your bday!!!
Thank you Chu Wei :)

5. (i) $log_5(x-1) - log_5(x+1) = 1 + log_5\frac{1}{7}$

$$\frac{log_5(x-1)}{log_5(x+1)} = log_5 5 + log_5\frac{1}{7} = log_5\frac{5}{7}$$

$$\frac{x-1}{x+1} = \frac{5}{7}$$

$$7x - 7 = 5x + 5$$

$$x = 6$$

(ii) $log_y 100 = lg\, y$

$$\frac{lg100}{lg\, y} = lg\, y$$

$$2 = (lg\, y)^2$$

$$\pm\sqrt{2} = lg\, y$$

$$y = 10^{-\sqrt{2}} \quad \text{or} \quad 10^{\sqrt{2}}$$

$$= 0.039 \quad \text{or} \quad 26$$

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

Suggested Answers

6. (i) $y = 9x^2 + (2m+1)x + 1 + c$ ----- (1)

$y = mx + c$ ----- (2)

(1) – (2)

$0 = 9x^2 + (2m+1)x - mx + 1$

$0 = 9x^2 + (m+1)x + 1$

$b^2 - 4ac = 0$

$(m+1)^2 - 4(9)(1) = 0$

$m^2 + 2m - 35 = 0$

$(m-5)(m+7) = 0$

$\therefore m = 5$ or -7(rej)

(ii) $y = 9x^2 + 11x + 1 + c$

Sub in (-2, 19)

$19 = 9(-2)^2 + 11(-2) + 1 + c$

$c = 4$

$\Rightarrow y = 9x^2 + 11x + 5$

$9x^2 + 6x + 1 = 0$

$(3x+1)^2 = 0$

$x = -\frac{1}{3},\ y = \frac{7}{3}$

$\therefore$ Coordinate of $P\left(-\frac{1}{3}, \frac{7}{3}\right)$

(iii) Line L is a vertical line, i.e. $x = a$, where $a \in \mathbb{R}$.

Solutions serve as asuggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

7. (ai) $P = 100e^{-kt}$

$t = 0,\ P = 100\%$

$t = 5730,\ P = 50 = 100e^{-k(5730)}$

$\Rightarrow \frac{1}{2} = e^{-5730k}$

$0.69314718 = 5730k$

$k = 0.00012096 \approx 0.000121$

(aii) $P = 100e^{-(0.00012096)(8000)} = 37.993\% \approx 38.0\%$

(b) $S = lg\frac{I}{c}$

Event 1	Event 2
$S_1 = S_1$	$S_2 = 2.4$
$I_1 = 50i$	$I_2 = i$

$\Rightarrow S_1 = lg\frac{50i}{c}$ ----- (1)

$\Rightarrow 2.4 = lg\frac{i}{c}$

$10^{2.4} = \frac{i}{c}$

$c = \frac{i}{10^{2.4}}$ ----- (2)

Sub (2) in (1)

$$S_1 = lg\frac{50i}{\frac{i}{10^{2.4}}} = lg\left(50i \times \frac{10^{2.4}}{i}\right) = lg\left(50 \times 10^{2.4}\right) = 4.098 \approx 4.1$$

Event S is 4.1.

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

Suggested Answers

8. (a) $y = ln(3x-1)$

$$\frac{dy}{dx} = \frac{3}{3x-1}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$0.06 = \frac{3}{20} \times \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{2}{5} \text{ units/s}$$

(bi) $y = 8-(2x+1)^3$

$$\frac{dy}{dx} = -3(2x+1)^2(2) = -6(2x+1)^2$$

$$-6(2x+1)^2 = 0$$

$$4x^2 + 4x + 1 = 0$$

$$b^2 - 4ac = 16 - 4(4)(1) = 0$$

$\therefore$ There is only 1 solution to $4x^2 + 4x + 1 = 0$.

Hence the curve $y = 8-(2x+1)^3$ has only 1 stationary point.

$\frac{dy}{dx} = -6(2x+1)^2 < 0$ for all $x \in \Re,\ x \neq -\frac{1}{2}$.

Since the value of $\frac{dy}{dx}$ remains negative for all x except at its stationary point $x = -\frac{1}{2}$, then this stationary point is a point of inflexion.

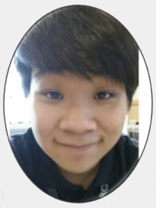
(bii) Coordinate of stationary point $\left(-\frac{1}{2},\ 8\right)$

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

Blk 283, Bishan Street 22, #01-185, S(570283) Chu Wei – 9077 9077 www.aostudies.com.sg

Hear what our students have to say…

Marcus Yeo Xin Hao,
Yishun Junior College



I joined Chuwei's tuition in my J2 year as i have heard of him from words of mouth from my friend and how they have benefited from his tuition, i was skeptical at first because i believed that with much practice, i would be able to conquer math myself, but never did i understood the difficulty of college math until i reach J2. i managed to pass promos with a stroke of luck with totally no understanding of J1's topic in J2 and struggled badly , That was when i decided to enroll in his tuition to give myself a chance to do well in math. From his torturous and monstrous self-made tutorial question, i had alot of trouble doing it, but he took the extra mile and explain preciously to me with the basic understanding of math. That was when i improve alot from being a C-D student to getting my A in Alevel. Math does not only required practice, it requires lots of patience and pain in doing challenging question to be able to challenge yourself with every tricky and new question you face. Thanks alot ChuWei for your help !

Ng Jing Ting,
Nanyang Junior College

AO Studies is a place with the most dedicated teachers. My awesome math tutor, Chu Wei just doesnt seem to leave his whatsapp conversations with us, endlessly solving our math questions even if it meant sacrificing his nights and midnights. Moreover, Chu Wei convicts to ensure his students would not leave lessons without learning anything, hence we see ourselves staying back completing his assignments most of the time.
Tuition can sometimes be a dreadful event, but strangely I love solving Chu Wei's monstrous tutorial questions! For this meant i have already mastered my concepts and not be afraid of taking up any school exams and finally the "A" Levels. True enough, I am proud to say I have never failed any math paper after the guidance of Chu Wei, when in fact math was my most hated subject since primary school. I even scored distinction in my A Levels!!!
There are many more fun-loving moments during lessons which made me loved attending tuition classes compared to the dull lessons in school.
Such fantastic environment is hard to come by, I didnt regret my choice.

9. (i) $\text{Grad}_{AB} = \dfrac{p-1}{2}$

$\text{Grad}_{CB} = 3-p$

Since angleCBO = angle ABO

$\text{Grad}_{AB} = -\text{Grad}_{CB}$

$\dfrac{p-1}{2} = -3-p$

$p = 5$ (shown)

(ii) $\text{Grad}_{AB} = 2$

$line_{AB}$: $y = 2x+1$ -------- (1)

$line_{AD}$: $y = -\dfrac{1}{2}x$ -------- (2)

Solve (1) and (2)

$x = -\frac{2}{5}$ and $y = \frac{1}{5}$

coordinates D $\left(-\frac{2}{5}, \frac{1}{5}\right)$

(iii) Area of Trapezium : $\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 & -2/5 & -2 \\ 1 & 5 & 3 & 1/5 & 1 \end{vmatrix} = \frac{34}{5}$ units^2

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

10. (i) (a) Amplitude = 2

Period = 360°

(b) Amplitude = 1

Period = 180°

(ii) $2\sin x + 1 = -\cos 2x$

$2\sin x + 1 = -\left(1 - 2\sin^2 x\right)$

$\sin^2 x - \sin x - 1 = 0$

$\sin x = \frac{1 \pm \sqrt{1 - 4(-1)}}{2}$

$\sin x = 1.618$ (rejected)

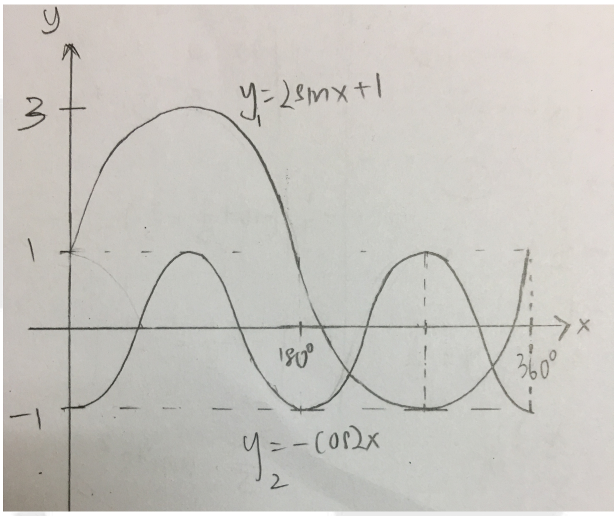
$\sin x = -0.618$

$\alpha = 38.2^\circ$

$x = 218.2^\circ, 321.8^\circ$

Blk 283, Bishan Street 22, #01-185, S(570283) Chu Wei – 9077 9077 www.aostudies.com.sg

(iii)



(iv) For $y_1 - y_2 > 0$

$y_1 > y_2$

$0 \le x < 218.2°$ or $321.8° < x \le 360°$

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.

11 (i) $4\cos\theta$ km

$4\sin\theta$ km

(ii) AreaΔOXY=AreaΔOYP+AreaΔOXP

$15 = 3(4\sin\theta) + 5(4\cos\theta)$

$15 = 12\sin\theta + 20\cos\theta$ (Shown)

(iii) $12\sin\theta + 20\cos\theta = R\cos(\theta - \alpha)$

$= R\cos\alpha\cos\theta + R\sin\alpha\sin\theta$

$R\cos\alpha = 20$

$R\sin\alpha = 12$

$\Rightarrow \tan\alpha = \dfrac{12}{20}$ $\alpha = 30.96°$

$\therefore 12\sin\theta + 20\cos\theta = \sqrt{544}\cos(\theta - 31.0°)$

(iii) $\sqrt{544}\cos(\theta - 31.0°) = 15$

$\theta = 80.9°$

Solutions serve as a suggestion only. All solutions are provided by the teachers from AO Studies. MOE / UCLES bears no responsibility for these suggested answers.