Suggested Answers

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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

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CENTRE NUMBER: S

INDEX NUMBER:

[redacted]

ADDITIONAL MATHEMATICS

Paper 2

4049/02

October/November 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Suggested Answers

ormulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2A+\cos^2A=1$$

$$\sec^2A=1+\tan^2A$$

$$\operatorname{cosec}^2A=1+\cot^2A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2A-\sin^2A=2\cos^2A-1=1-2\sin^2A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

Suggested Answers

1 Show that the equation $3e^x + 5 = 2e^{-x}$ has only one solution and find its value correct to 2 significant figures. [5]

$$3e^x + 5 = \frac{2}{e^x}$$

let $e^x = w$

$$3w + 5 = \frac{2}{w}$$

$$3w^2 + 5w - 2 = 0$$

$$(3w-1)(w+2) = 0$$

$w = \frac{1}{3}$ or $w = -2$

$e^x = \frac{1}{3}$ $\quad$ $e^x = -2$

rej $\therefore$ eqn $3e^x + 5 = 2e^{-x}$ only has 1 soln

$x = \ln\left(\frac{1}{3}\right)$

$= -1.1$ //

Suggested Answers

2 Show that $x = -1$ is a solution of the equation $3x^3 + 4x^2 - x - 2 = 0$ and hence solve the equation completely. [5]

Sub in $x = -1$

LHS : $3(-1)^3 + 4(-1)^2 - (-1) - 2$

$= -3 + 4 + 1 - 2 = 0 = \text{RHS}$

$\therefore x = -1$ is a soln

$3x^3 + 4x^2 - x - 2 = (x+1)(3x^2 + Ax - 2)$

comparing coefficient of x

$-1 = -2 + A$

$\therefore A = 1$

$3x^3 + 4x^2 - x - 2 = (x+1)(3x^2 + x - 2)$

$= (x+1)(3x-2)(x+1)$

$\therefore x = -1$ or $2/3$ //

3 **(a)** Given that $y = \dfrac{x}{(2x+1)^{\frac{1}{2}}}$, show that $\dfrac{dy}{dx} = \dfrac{x+1}{(2x+1)^{\frac{3}{2}}}$. [4]

$$\frac{dy}{dx} = \frac{(2x+1)^{1/2} - \frac{1}{2}(2x+1)^{-1/2}(2)(x)}{2x+1}$$

$$= \frac{(2x+1)^{-1/2}(2x+1-x)}{2x+1}$$

$$= \frac{x+1}{(2x+1)^{3/2}}$$

(b) Hence find the value of $\displaystyle\int_0^4 \frac{x}{(2x+1)^{\frac{3}{2}}}\,dx$. [4]

$$\int_0^4 \frac{x+1}{(2x+1)^{3/2}}\,dx = \left[\frac{x}{(2x+1)^{\frac{1}{2}}}\right]_0^4$$

$$\int_0^4 \frac{x}{(2x+1)^{3/2}}\,dx = \frac{4}{3} - \int_0^4 \frac{1}{(2x+1)^{3/2}}\,dx$$

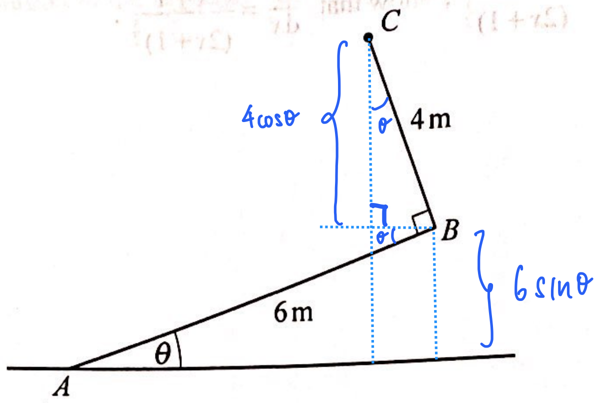
$$= \frac{4}{3} - \left[\frac{(2x+1)^{-1/2}}{(-\frac{1}{2})(2)}\right]_0^4$$

$$= \frac{4}{3} - \left[-\frac{1}{(2x+1)^{1/2}}\right]_0^4$$

$$= \frac{4}{3} - \left[-\frac{1}{3} - (-1)\right]$$

$$= \frac{2}{3}$$

4



The diagram shows two rods AB and BC rigidly joined at B so that angle $ABC = 90°$. The lengths of AB and BC are 6 m and 4 m respectively. A light is positioned at C. The point A is fixed on horizontal ground and the rod AB rotates in a vertical plane with the rod AB inclined at an angle θ to the ground.

(a) Show that the height, h metres, of C above the ground is given by

$$h = 6\sin\theta + 4\cos\theta.$$ [2]

vertical height of point B from ground : $6\sin\theta$

vertical height of point C to point B : $4\cos\theta$

$h = 6\sin\theta + 4\cos\theta$

(b) Express h in the form $R\sin(\theta+\alpha)$, where $R > 0$ and $0° \leqslant \alpha \leqslant 90°$. [4]

$$6\sin\theta + 4\cos\theta = R\sin(\theta+\alpha)$$
$$= R(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$$
$$= (R\cos\alpha)\sin\theta + (R\sin\alpha)\cos\theta$$

$\left.\begin{array}{l} R\cos\alpha = 6 \\ R\sin\alpha = 4 \end{array}\right\}$ $\tan\alpha = \frac{4}{6}$, $\alpha = 33.7°$

$R = \sqrt{4^2+6^2} = 2\sqrt{13}$

$\therefore h = 2\sqrt{13}\sin(\theta + 33.7°)$

(c) Find the value of h and the corresponding value of θ if the light at C is to be positioned as high as possible. [3]

$h_{max} = 2\sqrt{13}$

$\theta_{max} = 56.3^{\circ}$

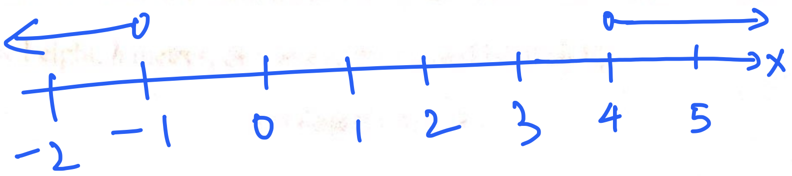
5 The equation of a curve is $y = 2x^2 - 6x + 3$.

(a) Find the set of values of x for which the curve lies above the line $y = 11$ and represent this set on a number line. [4]

$2x^2 - 6x + 3 > 11$

$2x^2 - 6x - 8 > 0$

$(2x - 8)(x + 1) > 0$



$\{x : x \in \mathbb{R},\ x < -1 \text{ or } x > 4\}$

Suggested Answers

The line $y = 2x + k$ is a tangent to the curve at the point P.

(b) Find the value of the constant k. [3]

$2x^2 - 6x + 3 = 2x + k$

$2x^2 - 8x + 3 - k = 0$

$b^2 - 4ac = 0$

$64 - 4(2)(3-k) = 0$

$64 - 24 + 8k = 0$

$8k = -40$

$k = -5$

(c) Find the coordinates of P. [2]

$2x^2 - 8x + 8 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$\therefore x = 2, \quad y = -1$

$P(2, -1)$ #

6 (a) Find the terms in $\frac{1}{x^2}$ and $\frac{1}{x^3}$ in the expansion of $\left(2-\frac{3}{x}\right)^6$. [4]

Term $\frac{1}{x^2}$: ${}^6C_2\,(2)^4\left(-\frac{3}{x}\right)^2$

$= 2160\,x^{-2}$

Term $\frac{1}{x^3}$ = ${}^6C_3\,(2^3)\left(-\frac{3}{x}\right)^3$

$= -4320\,x^{-3}$ ⌗

(b) Given that there is no term in $\frac{1}{x}$ in the expansion of $(x^2+ax)\left(2-\frac{3}{x}\right)^6$, find the value of the constant a. [2]

Coefficient $\frac{1}{x}$ in $(x^2+ax)\left(2-\frac{3}{x}\right)^6$

$: (1)(-4320)+(2160)(a)$

$\therefore\ 2160a = 4320$

$a = 2$ #

(c) Using the value of a found in part **(b)**, find the coefficient of x in the expansion of $(x^2+ax)\left(2-\frac{3}{x}\right)^6$. [3]

$(x^2+2x)\left[2^6+\binom{6}{1}(2)^5\left(-\frac{3}{x}\right)^1+\ldots\right]$

coefficient of $x = -576+2^7$

$= -448$ #

7 (a) A formula for working out the stopping distance, d, for a vehicle travelling at a speed v, is $d = av^2 + bv$, where a and b are constants. Values of d for different values of v have been collected. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [4]

$$\underbrace{\frac{d}{v}}_{Y} = \underbrace{a}_{m}\underbrace{v}_{X} + \underbrace{b}_{C}$$

let $\frac{d}{v}$ represent vertical axis

let v represent horizontal axis

Gradient of the graph = a

y-intercept of the graph = b

(b) Since 1960, the tiger population in an Asian country has been steadily decreasing. The table shows the estimated number of tigers, n, in the decades following 1960.
The decade 1960–1969 is taken as $t = 1$, and so on.

Year	1960–1969	1970–1979	1980–1989	1990–1999
Value of t	1	2	3	4
Number of tigers n	810	450	240	135
$\ln n$	6.697	6.109	5.481	4.905

A wild-life expert believed that these figures can be modelled by the formula $n = ab^t$, where a and b are constants.

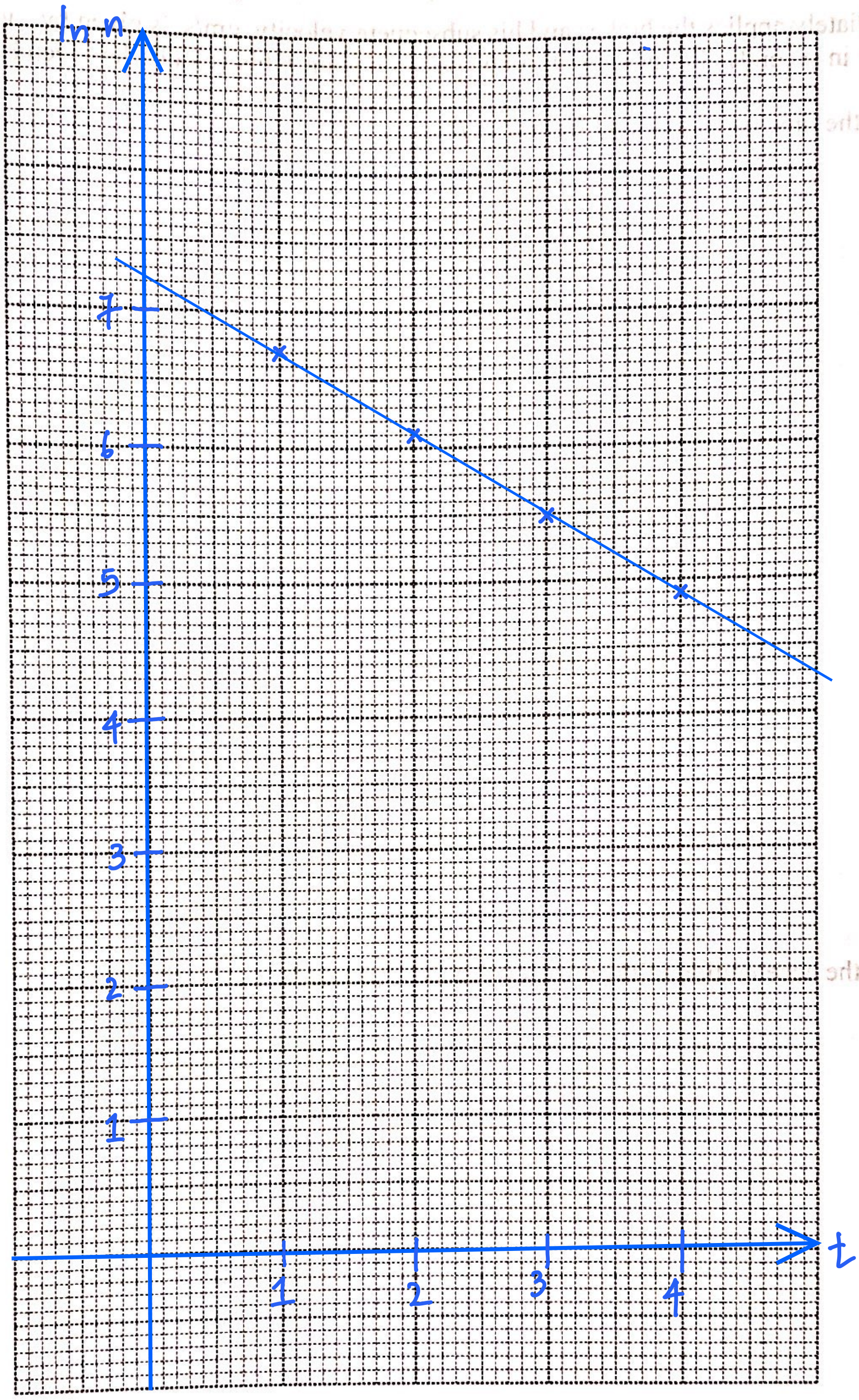
$n = ab^t$

$\ln n = \ln(ab^t)$

$\ln n = \ln a + t \ln b$

Suggested Answers

(i) Draw a straight line graph to show that the model is reasonable. [4]



(ii) Estimate the number of tigers in the decade 2000–2009. [2]

$t = 5$ $\quad \ln n = 4.3$

$n = 73.7$

(iii) Give a reason why this model might not be accurate in later decades. [1]

Extrapolation is required, data is only valid from 1960 – 1999.

More deforestation is possible in the later decades, number of tigers may decrease more than predicted.

Suggested Answers

8 A motorcyclist, travelling at a constant velocity of V m/s, passes a point A and sees roadworks ahead. He immediately applies the brakes and his subsequent velocity, v m/s, is given by $v = 24e^{\frac{-t}{6}}$, where t is the time in seconds after passing A. As he passes a point B his velocity has been halved.

(a) Find the time taken to travel from A to B. [3]

$t=0$, $V=24$ (initial vel)

$12 = 24e^{-t/6}$

$\frac{1}{2} = e^{-t/6}$

$-0.6931 = -t/6$

$t = 4.158$

≈ 4.16 seconds

(b) Find the acceleration of the motorcyclist as he passes B. [3]

$V = 24e^{-\frac{1}{6}t}$

$a = \frac{dv}{dt} = -4e^{-\frac{1}{6}t}$

acceleration at $B = -4e^{-\frac{1}{6}(4.158)}$

$= -4e^{\ln(\frac{1}{2})}$

$= -2\text{m/s}^2$ #

(c) Find the distance AB. [4]

$$\int_0^{4.158} v \, dt$$

$$= \int_0^{4.158} 24e^{-\frac{t}{6}} \, dt$$

$$= 24\left[-6e^{-t/6}\right]_0^{4.158}$$

$$= 24\left[-6\left(\tfrac{1}{2}\right) - (-6)\right]$$

$$= 72 \text{ m}$$ *

9 The equation of a circle is $(x+2a)^2+(y-a)^2=ka^2$, where a and k are positive constants. It is given that $k=4$.

(a) Explain why the y-axis is a tangent to the circle. [3]

horizontal line passes through centre $(-2a, a)$ cuts the y-axis at $(0, a)$, which is the radius of the circle

sub in $(0,a)$

LHS: $(2a)^2-(a-a)^2=4a^2=$ RHS

since radius $\perp$ tangent, y-axis is tangent to the circle.

(b) Find, in terms of a, the coordinates of the points on the circle at which the tangent to the circle is parallel to the x-axis. [2]

By observation, points are $(-2a,-a)$ and $(-2a, 3a)$

Suggested Answers

It is now given that $k = 5$.

(c) Verify that the circle passes through the origin O. [1]

$(x+2a)^2 + (y-a)^2 = 5a^2$

sub in $(0,0)$

LHS: $(2a)^2 + (-a)^2 = 5a^2 = $ RHS

$\therefore$ circle passes through origin O.

(d) Given that OP is a diameter of the circle, find, in terms of a, the coordinates of the point at which the tangent to the circle at P meets the x-axis. [6]

By similar $\triangle$, point $P\ (-4a, 2a)$

Grad of $OP = -\frac{1}{2}$

Grad of tangent at $P = 2$

Eq^{n} of tangent $y - 2a = 2(x+4a)$

$y = 2x + 10a$

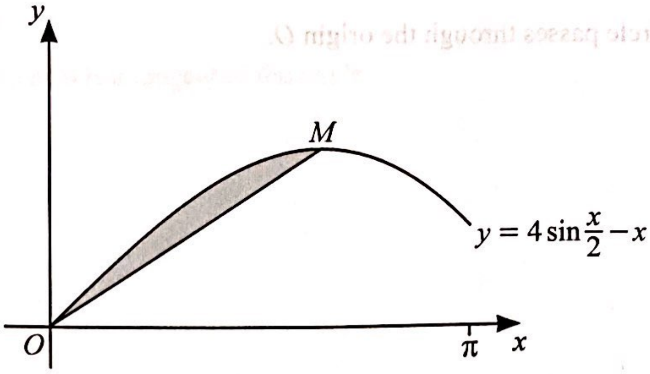
when $y=0$ $\quad 2x = -10a$

$x = -5a$

coordinate : $(-5a, 0)$ #

Suggested Answers

10



The diagram shows the curve $y = 4\sin\frac{x}{2} - x$ for $0 \leqslant x \leqslant \pi$ radians. The point M is the maximum point of the curve and OM is a straight line.

Show that the area of the shaded region is $4 - \frac{2\pi\sqrt{3}}{3}$ units^2. [12]

$$\frac{dy}{dx} = 2\cos\frac{x}{2} - 1 = 0$$

$$\cos\frac{x}{2} = \frac{1}{2}$$

$$x = \frac{2\pi}{3} \qquad y = 4\sin\frac{\pi}{3} - \frac{2\pi}{3} = 2\sqrt{3} - \frac{2\pi}{3}$$

$$\text{Req. Area} = \int_0^{2\pi/3} 4\sin\frac{x}{2} - x \, dx - \frac{1}{2}\left(\frac{2\pi}{3}\right)\left(2\sqrt{3} - \frac{2\pi}{3}\right)$$

$$= \left[-8\cos\frac{x}{2} - \frac{x^2}{2}\right]_0^{\frac{2\pi}{3}} - \frac{\pi}{3}\left(2\sqrt{3} - \frac{2\pi}{3}\right)$$

$$= \left(-4 - \frac{2\pi^2}{9}\right) - (-8) - \frac{2\sqrt{3}\pi}{3} + \frac{2\pi^2}{9}$$

$$= 4 - \frac{2\pi\sqrt{3}}{3} \text{ units}^2$$

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Suggested Answers

Continuation of working space for question 10.

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